THEORETICAL FOUNDATIONS FOR THE ANALYSIS OF LAMINAR AND TURBULENT SPRAY FLOWS

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GOALS

- Unify averaging processes for two-phase flow, LES, and computation.
- Evaluate new flux and source terms.
- Identify needs for modelling of spray microstructure (sub-grid, high wavenumber)

RESOLUTION

- In typical spray problem, there are many droplets in the volume that we are able to resolve numerically; so averaging over a small volume is performed.
- LES analysis also involves similar type of averaging or filtering.
- Discretization associated with computation is a form of averaging.
- The poorest resolution is always determining; so there is no reason to seek improved resolution in only two of the three processes.
- Therefore, unification would be helpful.

Existing Literature and New Needs

- **There is substantial analysis for two-phase flow averaging methods addressing particle-laden, bubble-laden, and porous flows; suspensions; and sprays.**
- **Burning fuel spray flows have some special features that require special analytical treatment:**

- Although liquid volume fraction is small, the fractions of mass, momentum, and energy are significant;

- Fast vaporization so droplets and gas are not in kinematic or thermal equilibrium;

- Droplets can have high Re and Pe numbers, so large gradients can exist within the microstructure;

- Vaporization rates depend on internal droplet mechanics and transport; and

- The smallest turbulent scales can have length scales comparable to droplet size and spacing.

Primitive Continuity and Momentum Equations

$$\frac{\partial(\rho Y_n)}{\partial t}+\frac{\partial(\rho Y_n u_j)}{\partial x_j}+\frac{\partial(\rho Y_n V_{n,j})}{\partial x_j}=\rho\omega_n\ , \qquad n=1,\cdots,N$$

$$V_{n,i}=\frac{D_n}{Y_n}\frac{\partial Y_n}{\partial x_i}$$

$$\frac{\partial\rho}{\partial t}+\frac{\partial(\rho u_j)}{\partial x_j}=0$$

$$\frac{\partial(\rho u_i)}{\partial t}+\frac{\partial(\rho u_i u_j)}{\partial x_j}+\frac{\partial p}{\partial x_i}=\frac{\partial\tau_{ij}}{\partial x_j}+\rho g_i$$

Primitive Energy Equation

$$\frac{\partial(\rho h)}{\partial t} + \frac{\partial(\rho h u_j)}{\partial x_j} + \frac{\partial q_j}{\partial x_j} = \frac{\partial p}{\partial t} + u_j \frac{\partial p}{\partial x_j} + \Phi + \sum_{n=1}^{N} \rho \omega_n Q_n$$

$$\Phi = \tau_{ij} \frac{\partial u_i}{\partial x_j}$$

$$q_i = -\lambda \frac{\partial T}{\partial x_i} + q_{rad,i} + \sum_{n=1}^{N} \rho V_{n,i} h_n Y_n$$

These primitive equations apply to both the gas and liquid (continuous and discrete phases). For the gas, we have a state equation:

$$p = \rho R T$$

Weighting Function and Averaging Volume

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} G(\vec{x}-\vec{\xi})\,d\vec{\xi} = 1$$

$$d\vec{\xi} = d\xi_1 d\xi_2 d\xi_3 .$$

The weighting function G depends only on the relative position.

Many choices are possible for G; three symmetric choices are shown.

$$G = \frac{1}{2}\left(\frac{b}{\pi}\right)^{3/2} e^{-b|\vec{x}-\vec{\xi}|^2}$$

$$G = \begin{cases} \frac{3}{4\pi a^3} & \text{if } 0 \leq |\vec{x}-\vec{\xi}| \leq a; \\ 0 & \text{if } |\vec{x}-\vec{\xi}| > a. \end{cases}$$

$$G = \begin{cases} \frac{1}{8abc} & \text{if } -a \leq x_1 \leq a\,;\ -b \leq x_2 \leq b\,;\ -c \leq x_3 \leq c\,; \\ 0 & \text{if } \quad otherwise. \end{cases}$$

Symmetry in the G and therefore in the averaging volume is not necessary but usually assumed. The shape, size, and orientation of the averaging volume are assumed to remain uniform over the field.

Important Properties of G

$$\partial G/\partial x_i = -\partial G/\partial \xi_i.$$

$$G \to 0 \text{ as } \left|\vec{x}-\vec{\xi}\right| \to \infty.$$

Void Volume

$$\overline{\theta(\vec{x},t)} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} G(\vec{x}-\vec{\xi})\theta(\vec{\xi},t)d\vec{\xi}$$

$$\overline{1-\theta(\vec{x},t)} = 1-\overline{\theta(\vec{x},t)} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} G(\vec{x}-\vec{\xi})(1-\theta(\vec{\xi},t))d\vec{\xi}$$

Averaged Quantities

$$\overline{\rho(\vec{x},t)} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} G(\vec{x}-\vec{\xi})\theta(\vec{\xi},t)\rho(\vec{\xi},t)d\vec{\xi}$$

$$\overline{\rho(\vec{x},t)Y_n(\vec{x},t)} \stackrel{def}{=} \overline{\rho(\vec{x},t)}\ \widetilde{Y_n(\vec{x},t)} \stackrel{def}{=} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} G(\vec{x}-\vec{\xi})\theta(\vec{\xi},t)\rho(\vec{\xi},t)Y_n(\vec{\xi},t)d\vec{\xi}$$

Averaged Properties

$$\overline{\rho(\vec{x},t)h(\vec{x},t)} \stackrel{def}{=} \overline{\rho(\vec{x},t)}\ \widetilde{h(\vec{x},t)} \stackrel{def}{=} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} G(\vec{x}-\vec{\xi})\theta(\vec{\xi},t)\rho(\vec{\xi},t)h(\vec{\xi},t)d\vec{\xi}$$

and

$$\overline{\rho(\vec{x},t)\omega_n(\vec{x},t)} \stackrel{def}{=} \overline{\rho(\vec{x},t)}\ \widetilde{\omega_n(\vec{x},t)} \stackrel{def}{=} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} G(\vec{x}-\vec{\xi})\theta(\vec{\xi},t)\rho(\vec{\xi},t)\omega_n(\vec{\xi},t)d\vec{\xi}$$

$$\overline{\Phi(\vec{x},t)} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} G(\vec{x}-\vec{\xi})\theta(\vec{\xi},t)\Phi(\vec{\xi},t)d\vec{\xi}$$

$$\overline{\rho(\vec{x},t)u_i(\vec{x},t)} \stackrel{def}{=} \overline{\rho(\vec{x},t)}\ \widetilde{u_i(\vec{x},t)} \stackrel{def}{=} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} G(\vec{x}-\vec{\xi})\theta(\vec{\xi},t)\rho(\vec{\xi},t)u_i(\vec{\xi},t)d\vec{\xi}$$

$$\overline{\theta(\vec{x},t)}\ \overline{p(\vec{x},t)} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} G(\vec{x}-\vec{\xi})\theta(\vec{\xi},t)p(\vec{\xi},t)d\vec{\xi}$$

More Averaged Properties

$$\overline{\rho(\vec{x},t)Y_n(\vec{x},t)u_i(\vec{x},t)} \stackrel{def}{=} \overline{\rho(\vec{x},t)} \;\langle Y_n(\vec{x},t)u_i(\vec{x},t)\rangle$$
$$\stackrel{def}{=} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} G(\vec{x}-\vec{\xi})\theta(\vec{\xi},t)\rho(\vec{\xi},t)Y_n(\vec{\xi},t)u_i(\vec{\xi},t)d\vec{\xi}$$

$$\overline{\rho(\vec{x},t)Y_n(\vec{x},t)V_{n,i}(\vec{x},t)} \stackrel{def}{=} \overline{\rho(\vec{x},t)} \;\langle Y_n(\vec{x},t)V_{n,i}(\vec{x},t)\rangle$$
$$\stackrel{def}{=} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} G(\vec{x}-\vec{\xi})\theta(\vec{\xi},t)\rho(\vec{\xi},t)Y_n(\vec{\xi},t)V_{n,i}(\vec{\xi},t)d\vec{\xi}$$

$$\overline{\rho(\vec{x},t)h(\vec{x},t)u_i(\vec{x},t)} \stackrel{def}{=} \overline{\rho(\vec{x},t)} \;\langle h(\vec{x},t)u_i(\vec{x},t)\rangle$$
$$\stackrel{def}{=} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} G(\vec{x}-\vec{\xi})\theta(\vec{\xi},t)\rho(\vec{\xi},t)h(\vec{\xi},t)u_i(\vec{\xi},t)d\vec{\xi}$$

$$\overline{q_i(\vec{x},t)} \stackrel{def}{=} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} G(\vec{x}-\vec{\xi})\theta(\vec{\xi},t)q_i(\vec{\xi},t)d\vec{\xi}$$

$$\overline{\rho(\vec{x},t)u_i(\vec{x},t)u_j(\vec{x},t)} \stackrel{def}{=} \overline{\rho(\vec{x},t)} \;\langle u_i(\vec{x},t)u_j(\vec{x},t)\rangle$$
$$\stackrel{def}{=} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} G(\vec{x}-\vec{\xi})\theta(\vec{\xi},t)u_i(\vec{\xi},t)u_j(\vec{\xi},t)d\vec{\xi}$$

$$\overline{\theta(\vec{x},t)}\;\overline{\tau_{ij}(\vec{x},t)} \stackrel{def}{=} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} G(\vec{x}-\vec{\xi})\theta(\vec{\xi},t)\tau_{ij}(\vec{\xi},t)d\vec{\xi}$$

Average of Derivatives

$$\overline{\frac{\partial \phi(\vec{x},t)}{\partial t}} = \frac{\partial \overline{\phi(\vec{x},t)}}{\partial t} + \int_S \int G(\vec{x}-\vec{\zeta})\phi(\vec{\zeta},t)u_{\theta,j}(\vec{\zeta},t)dA_j$$

$$\overline{\frac{\partial \phi(\vec{x},t)}{\partial x_i}} = \frac{\partial \overline{\phi(\vec{x},t)}}{\partial x_i} - \int_S \int G(\vec{x}-\vec{\zeta})\phi(\vec{\zeta},t)dA_i$$

Effects of Changes in Size, Shape, and/or Orientation Of Averaging Volume

$$\overline{\frac{\partial \phi(\vec{x},t)}{\partial x_i}} = \frac{\partial \overline{\phi(\vec{x},t)}}{\partial x_i} + \frac{\partial(log(V))}{\partial x_i}\overline{\phi} - \int_A \int \phi(\vec{\eta})\theta(\vec{\eta})\frac{1}{V}\frac{\partial n(\vec{\eta})}{\partial x_i}dA^* - \int_S \int G(\vec{x}-\vec{\zeta})\phi(\vec{\zeta},t)dA_i$$

Averaged Gas-phase Continuity And Species Continuity Equations

$$\frac{\partial \overline{\rho}}{\partial t} + \frac{\partial (\overline{\rho}\, \tilde{u}_j)}{\partial x_j} = \dot{M} \quad \stackrel{def}{=} \int_S \int G(\vec{x} - \vec{\zeta})\rho(\vec{\zeta}, t)[u_j(\vec{\zeta}, t) - u_{\theta,j}(\vec{\zeta}, t)]dA_j$$

$$\frac{\partial (\overline{\rho}\, \tilde{Y}_n)}{\partial t} + \frac{\partial (\overline{\rho}\, \tilde{Y}_n\, \tilde{u}_j)}{\partial x_j} + \frac{\partial (\overline{\rho}\, \tilde{Y}_n\, \tilde{V}_{n,j})}{\partial x_j} = \overline{\rho}\, \tilde{\omega}_n + \dot{M}\epsilon_n - \frac{\partial (\overline{\rho}\, \alpha_{n,j})}{\partial x_j} - \frac{\partial (\overline{\rho}\, \beta_{n,j})}{\partial x_j}$$

$$n = 1, \cdots, N$$

$$\alpha_{n,i} \stackrel{def}{=} \langle Y_n u_i \rangle - \tilde{Y}_n\, \tilde{u}_i \qquad \beta_{n,i} \stackrel{def}{=} \langle Y_n V_{n,i} \rangle - \tilde{Y}_n\, \tilde{V}_{n,i}$$

$$\dot{M}\epsilon_n = \int_S \int G(\vec{x} - \vec{\zeta})\rho(\vec{\zeta}, t)Y_n(\vec{\zeta}, t)[u_j(\vec{\zeta}, t) - u_{\theta,j}(\vec{\zeta}, t) + V_{n,j}(\vec{\zeta}, t)]dA_j$$

Averaged Momentum Equation

$$\frac{\partial(\,\overline{\rho}\,\tilde{u}_i\,)}{\partial t}+\frac{\partial(\,\overline{\rho}\,\tilde{u}_i\,\tilde{u}_j\,)}{\partial x_j}+\overline{\theta}\frac{\partial\,\overline{p}}{\partial x_i}=\overline{\theta}\frac{\partial\,\overline{\tau}_{ij}}{\partial x_j}+\overline{\rho}g_i-F_i+\dot{M}\overline{u}_{l,j}-\frac{\partial(\,\overline{\rho}\,\Gamma_{ij})}{\partial x_j}$$

$$F_i \;=\; \int_S\int G(\vec{x}-\vec{\zeta})\left\{[\tau_{ij}(\vec{\zeta},t)-\delta_{ij}p(\vec{\zeta},t)]-[\,\overline{\tau_{ij}(\vec{x},t)}-\delta_{ij}\overline{p(\vec{x},t)}\,]\right\}dA_j$$

$$\Gamma_{ij}\;\stackrel{def}{=}\;\langle u_i u_j\rangle-\tilde{u}_i\,\tilde{u}_j$$

Averaged Energy Equation

$$\frac{\partial(\bar{\rho}\,\tilde{h})}{\partial t} + \frac{\partial(\bar{\rho}\,\tilde{h}\,\tilde{u}_j)}{\partial x_j} + \frac{\partial\,\bar{q}_j}{\partial x_j} - \bar{\theta}\left\{\frac{\partial\,\bar{p}}{\partial t} + \tilde{u}_j\frac{\partial\,\bar{p}}{\partial x_j}\right\}$$
$$= \bar{\Phi} + \sum_{n=1}^{N}\bar{\rho}\,\tilde{\omega}_n\,Q_n + \dot{M}\,[\,\tilde{h}_{g,s} - L_{eff}]$$
$$+ S_5 + \bar{p}\tilde{u}_j\frac{\partial\bar{\theta}}{\partial x_j} + \Delta - \frac{\partial(\bar{\rho}\,E_j)}{\partial x_j}$$

$$S_5 = \int_S\int G(\vec{x}-\vec{\zeta})\left\{p(\vec{\zeta},t) - \overline{p(\vec{x},t)}\right\}u_{\theta,j}(\vec{\zeta},t)\,dA_j$$

$$\Delta \stackrel{def}{=} \overline{u_j\frac{\partial p}{\partial x_j}} - \tilde{u}_j\,\frac{\partial(\bar{\theta}\,\bar{p})}{\partial x_j}$$

$$E_i \stackrel{def}{=} \langle u_i h\rangle - \tilde{u}_i\,\tilde{h}$$

$$\bar{\theta}\,\bar{p} = \bar{\rho}\,\widetilde{RT} = \bar{\rho}\,[\,\tilde{h} - \tilde{e}\,]$$

AVERAGED ENTROPY

$$\rho\left\{\frac{\partial s}{\partial t}+u_j\frac{\partial s}{\partial x_j}\right\}=\frac{\partial(\rho s)}{\partial t}+\frac{\partial(\rho u_j s)}{\partial x_j}=\frac{1}{T}\left\{-\frac{\partial q_j}{\partial x_j}+\Phi+\sum_{n=1}^{N}\rho\omega_n Q_n\right\}\stackrel{def}{=}\frac{R_1}{T}$$

$$\frac{\partial(\,\bar{\rho}\,\tilde{s}\,)}{\partial t}+\frac{\partial(\,\bar{\rho}\,\tilde{u}_j\tilde{s}\,)}{\partial x_j}=\int_S\int G(\vec{x}-\vec{\zeta})(u_j-u_{\theta,j})\rho s\,dA_j-\frac{\partial(\,\bar{\rho}\,H_j)}{\partial x_j}+\frac{\overline{R_1}}{\tilde{T}}+J$$

$$\bar{\rho}\frac{\partial\tilde{s}}{\partial t}+\bar{\rho}\,\tilde{u}_j\frac{\partial\tilde{s}}{\partial x_j}=\dot{M}\left(\tilde{s}_{g,s}-\tilde{s}-\frac{L_{eff}}{\tilde{T}}\right)-\frac{\partial(\,\bar{\rho}\,H_j)}{\partial x_j}+\frac{R_2}{\tilde{T}}+J$$

$$\tilde{s}_{g,s}-\tilde{s}-\frac{L_{eff}}{\tilde{T}}=\frac{\tilde{h}_{g,s}-\tilde{h}-L_{eff}}{\tilde{T}}$$

$$\overline{R}_1=-\frac{\partial\,\bar{q}_j}{\partial x_j}+\overline{\Phi}+\sum_{n=1}^{N}\bar{\rho}\,\tilde{\omega}_n\,Q_n-\dot{M}L_{eff}\stackrel{def}{=}R_2-\dot{M}L_{eff}$$

$$H_i\stackrel{def}{=}\langle u_i s\rangle-\tilde{u}_i\;\tilde{s}$$

$$J\stackrel{def}{=}\overline{\left\{\frac{R_1}{T}\right\}}-\frac{\overline{R_1}}{\tilde{T}}$$

AVERAGED VORTICITY

The volume average of the curl of the velocity differs from the curl of the mass-averaged velocity

$$\overline{\omega}_i = \epsilon_{ijk}\overline{\frac{\partial u_j}{\partial x_k}} = \epsilon_{ijk}\frac{\partial\,\overline{u}_j}{\partial x_k} - \epsilon_{ijk}\int_S\int G(\vec{x}-\vec{\zeta})u_j(\vec{\zeta},t)dA_k$$
$$= \Omega_i + \epsilon_{ijk}\frac{\partial(\,\overline{u}_j - \tilde{u}_j)}{\partial x_k} - \epsilon_{ijk}\int_S\int G(\vec{x}-\vec{\zeta})u_j(\vec{\zeta},t)dA_k$$

$$\frac{\partial\Omega_i}{\partial t} + \tilde{u}_j\frac{\partial\Omega_i}{\partial x_j} = \Omega_j\frac{\partial\tilde{u}_i}{\partial x_j} - \Omega_i\frac{\partial\tilde{u}_j}{\partial x_j} - \epsilon_{ijk}\frac{\partial(\,\overline{\theta}/\overline{\rho}\,)}{\partial x_k}\frac{\partial\overline{p}}{\partial x_j}$$
$$+\epsilon_{ijk}\left\{\frac{\overline{\theta}}{\overline{\rho}}\frac{\partial^2\overline{\tau}_{jr}}{\partial x_k\partial x_r} + \frac{\partial(\,\overline{\theta}/\overline{\rho})}{\partial x_k}\frac{\partial\overline{\tau}_{jm}}{\partial x_m}\right\}$$
$$-\epsilon_{ijk}\left\{\frac{\partial F_j}{\partial x_k} + \dot{M}\left\{\frac{\partial\tilde{u}_j}{\partial x_k} - \frac{\partial\overline{u}_{l,j}}{\partial x_k}\right\} + \frac{\partial\dot{M}}{\partial x_k}\{\tilde{u}_j - \overline{u}_{l,j}\}\right\}$$
$$-\epsilon_{ijk}\frac{\partial^2(\,\overline{\rho}\,\Gamma_{jr})}{\partial x_k\partial x_r}$$

Droplet Equations in Lagrangian Form

$$\frac{d\bar{\theta}}{dt} = \frac{\dot{M}}{\rho_l}$$

$$\frac{dV_{drop}}{dt} = \frac{\dot{M}}{\rho_l n} = \frac{\dot{m}}{\rho_l}$$

$$\bar{\rho}_l \frac{d\overline{Y}_{l,n}}{dt} = -\dot{M}[\epsilon_{l,n} - \overline{Y}_{l,n}] - \frac{\partial(\bar{\rho}_l \, \alpha_{l,n,j})}{\partial x_j} \quad n = 1, \cdots, N$$

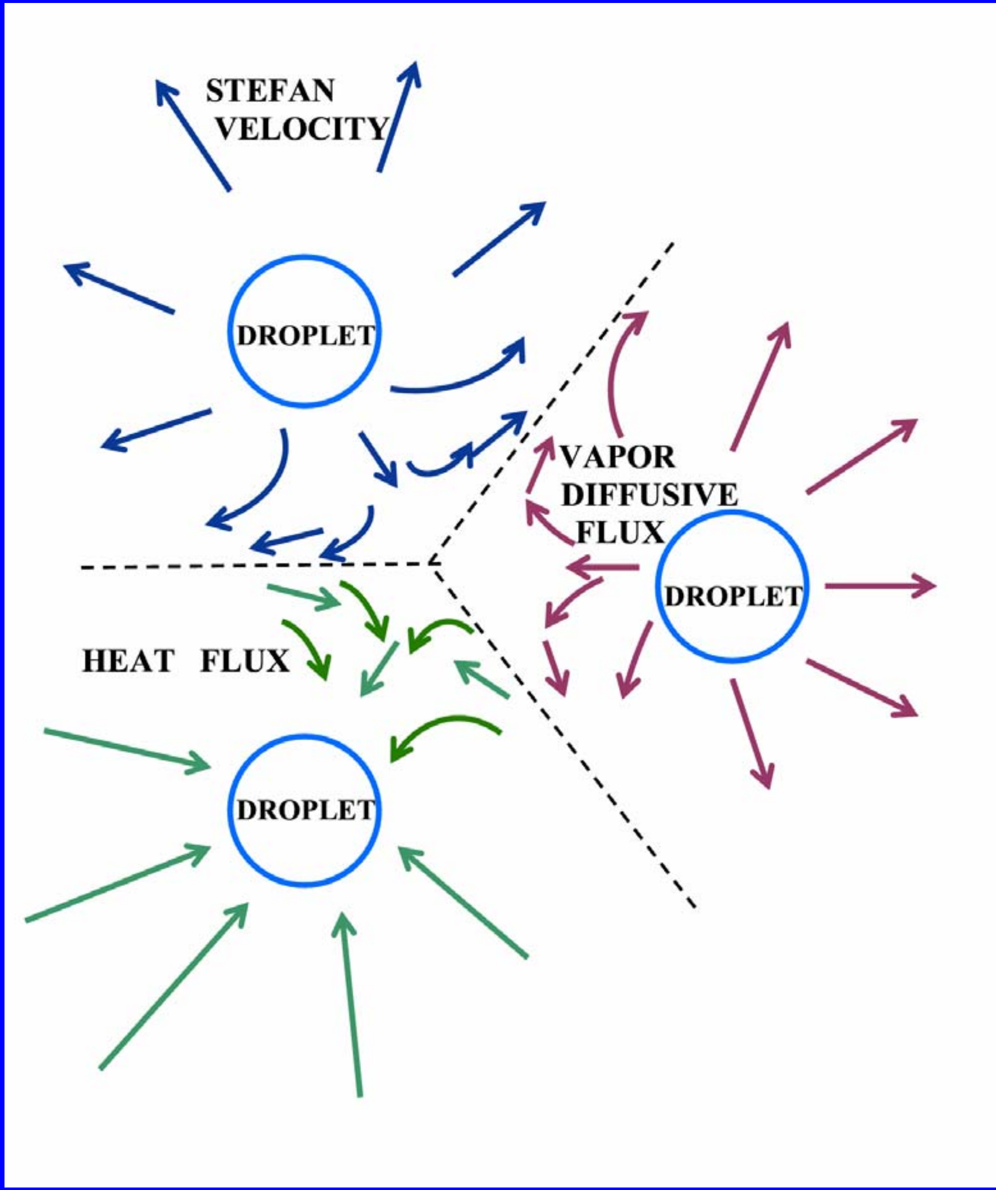
$$\begin{aligned} \bar{\rho}_l \frac{d\bar{u}_{l,i}}{dt} = & \frac{1-\bar{\theta}}{\bar{\theta}} \bar{\rho} \frac{D\,\tilde{u}_i}{Dt} + \left\{ \bar{\rho}_l - \bar{\rho} \frac{1-\bar{\theta}}{\bar{\theta}} \right\} g_i + \frac{F_i}{\bar{\theta}} \\ & - \dot{M} \frac{1-\bar{\theta}}{\bar{\theta}} [\, \tilde{u}_i - \bar{u}_{l,i} \,] - \frac{\partial(\, \bar{\rho}_l \, \Gamma_{l,ij})}{\partial x_j} + \frac{1-\bar{\theta}}{\bar{\theta}} \frac{\partial(\, \bar{\rho} \, \Gamma_{ij})}{\partial x_j} \end{aligned}$$

$$\bar{\rho}_l \frac{d\bar{h}_l}{dt} = (1 - \bar{\theta}) \frac{d\bar{p}_l}{dt} + \overline{\Phi}_l + \dot{Q}_l - S_{l,5} + \Delta_l - \frac{\partial(\, \bar{\rho}_l \, E_{l,j})}{\partial x_j}$$

Conservation of Droplet Numbers – neglecting coalescence and break-up

$$\frac{dn}{dt} = -n\frac{\partial \, \overline{u}_{l,j}}{\partial x_j}$$

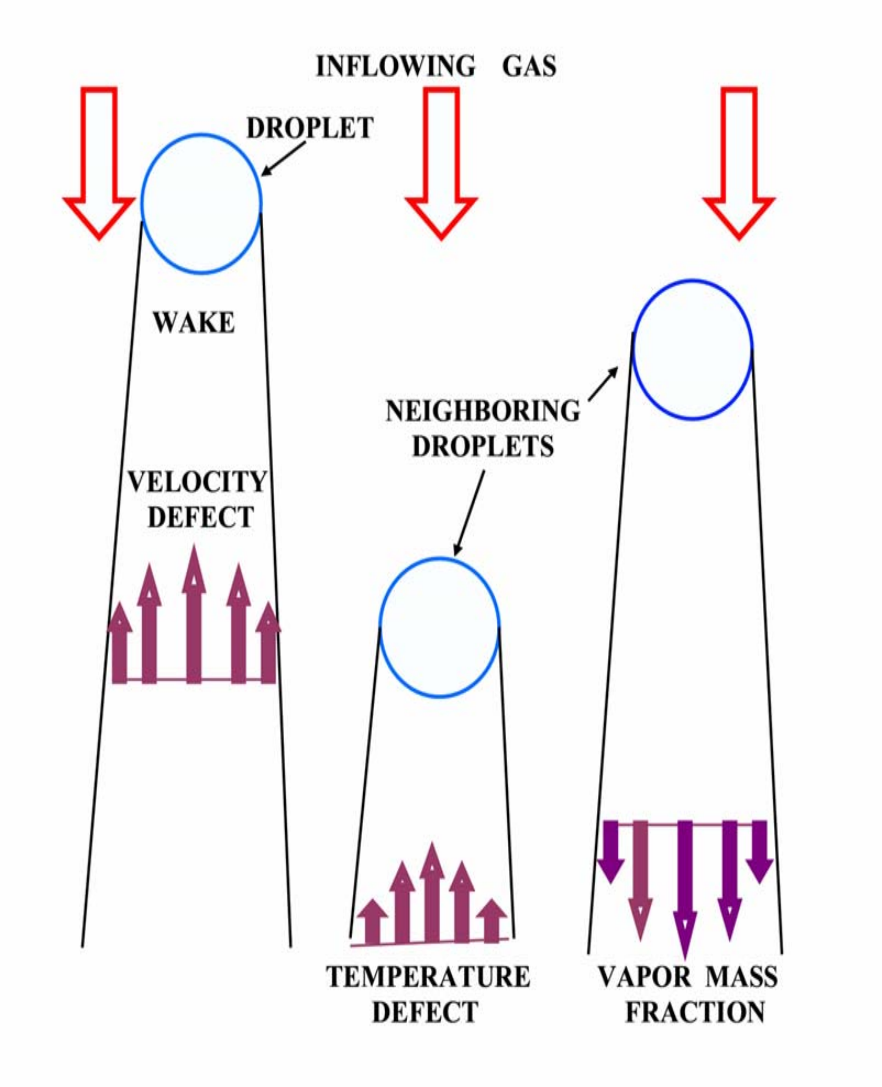
Dense Spray Vaporization



Lack of symmetry can result in significant values for the new fluxes: $\alpha_{n,i}$, $\beta_{n,i}$, Γ_{ij} , and E_i

Modelling of these terms requires some analysis of microstructure.

Convective Vaporization



$$\frac{u_1}{U_0} = \frac{C_D Re}{8x/R}\, exp\left\{-\frac{U_0 r^2}{4\nu x}\right\}$$

$$\frac{T_1}{T_0} = \frac{\dot{m} L_{eff}}{4\pi\rho c_p T_0 \nu x}\, exp\left\{-\frac{U_0 r^2}{4\nu x}\right\}$$

$$Y_F = \frac{\dot{m}}{4\pi\rho\nu x}\, exp\left\{-\frac{U_0 r^2}{4\nu x}\right\}$$

MICROSTRUCTURE AVERAGES AND PERTURBATIONS

$$\overline{u} = U_0 \left\{ 1 - \frac{\pi R^2}{A} \frac{C_D}{2} \right\}$$

$$u'(x,r) = u - \overline{u} = U_0 \left\{ \frac{\pi R^2}{A} \frac{C_D}{2} - \frac{C_D Re}{8x/R}\, exp \left\{ - \frac{U_0 r^2}{4 \nu x} \right\} \right\}$$

$$\overline{T} = T_0 \left\{ 1 - \frac{\dot{m} L_{eff}}{\rho U_0 A c_p T_0} \right\}$$

$$T'(x,r) = T - \overline{T} = \frac{\dot{m} L_{eff}}{\rho U_0 A c_p} \left\{ 1 - \frac{A}{\pi R^2} \frac{Re}{4x/R}\ \exp \left\{ - \frac{U_0 r^2}{4 \nu x} \right\} \right\}$$

$$\overline{Y}_F = \frac{\dot{m}}{\rho U_0 A}$$

$$Y'_F = \frac{\dot{m}}{\rho U_0 A} \left\{ \frac{A}{\pi R^2} \frac{Re}{4x/R}\, exp \left\{ - \frac{U_0 r^2}{4 \nu x} \right\} - 1 \right\}$$

Microstructure Product Averages

$$\frac{\Gamma_{11}}{\overline{u}\,\overline{u}} = \frac{\overline{u'u'}}{U_0 U_0} = \frac{C_D^2}{32}\frac{\pi R^2}{A} Re \frac{log(L/R)}{L/R}$$

$\Gamma_{11}/U_0U_0 = O(10^{-2})$
To $O(10^{-1})$
for $10 < Re < 100$

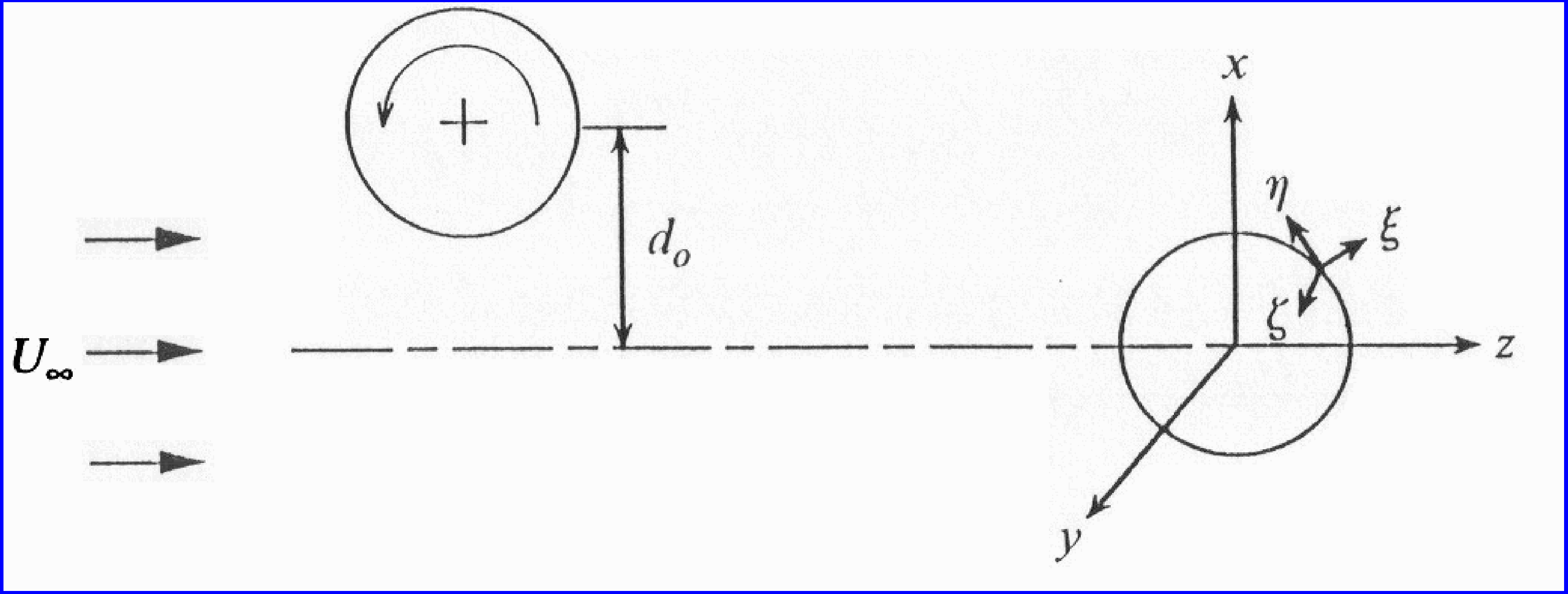
$$\frac{E_1}{\overline{u}\,\overline{h}} = \frac{\overline{u'T'}}{U_0 T_0} = \frac{2\dot{m}L_{eff}}{\pi R^2 \rho U_0 c_p T_0 C_D}\frac{\overline{u'u'}}{U_0^2}$$

$E_1 / U_0 c_p T_0 = O(10^{-1})$

$$\frac{\alpha_{F,1}}{\overline{u}\,\overline{Y}_F} = \frac{\overline{u'Y_F'}}{U_0 \overline{Y}_F} = -\frac{2}{C_D}\frac{A}{\pi R^2}\frac{\overline{u'u'}}{U_0^2}$$

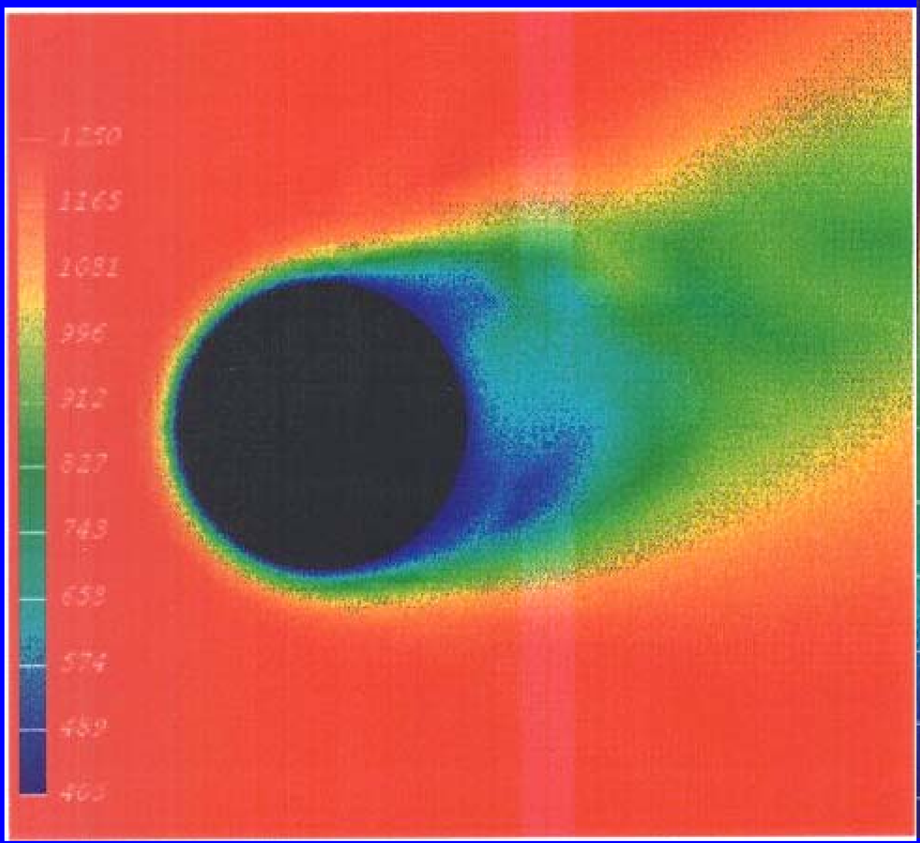
$\alpha_1 / U_0 \overline{Y}_F = O(10^{-1})$

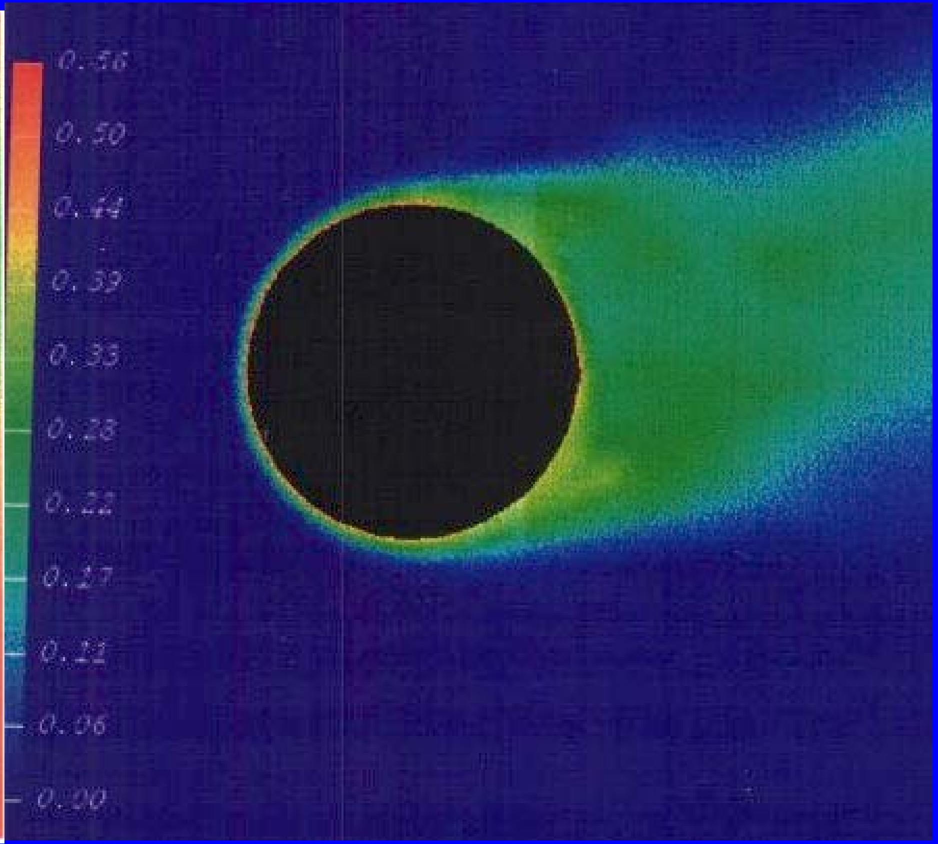
Vortex – Droplet Collision



A vortex flows towards the droplet. Positive and negative displacements d_o (clockwise and counter clockwise rotations), Reynolds number Re , vortex strength Γ , and vortex size σ are considered in a parameter survey using Navier-Stokes computations.

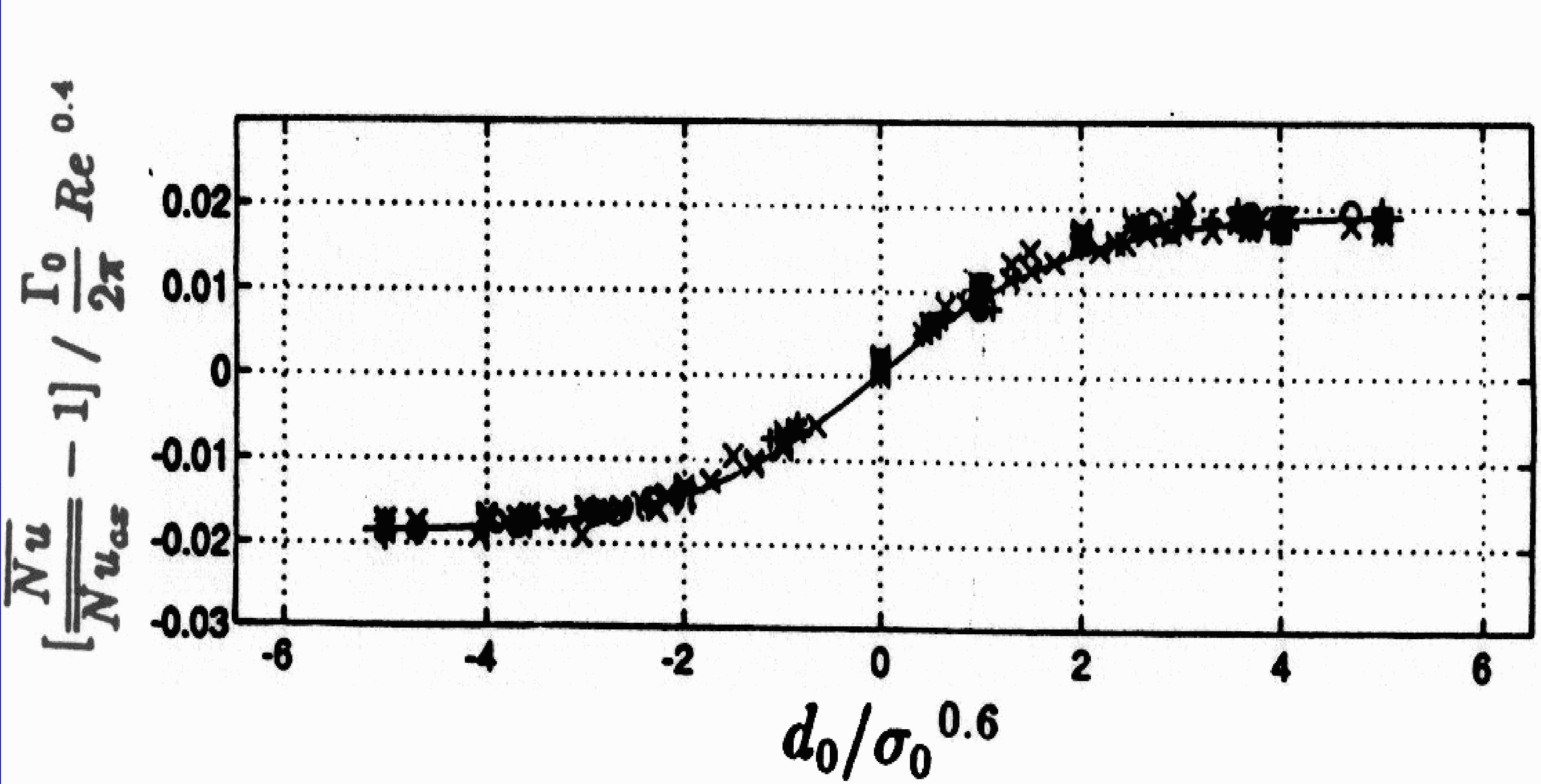
Temperature and Species Mass Fraction Variation During a Collision.





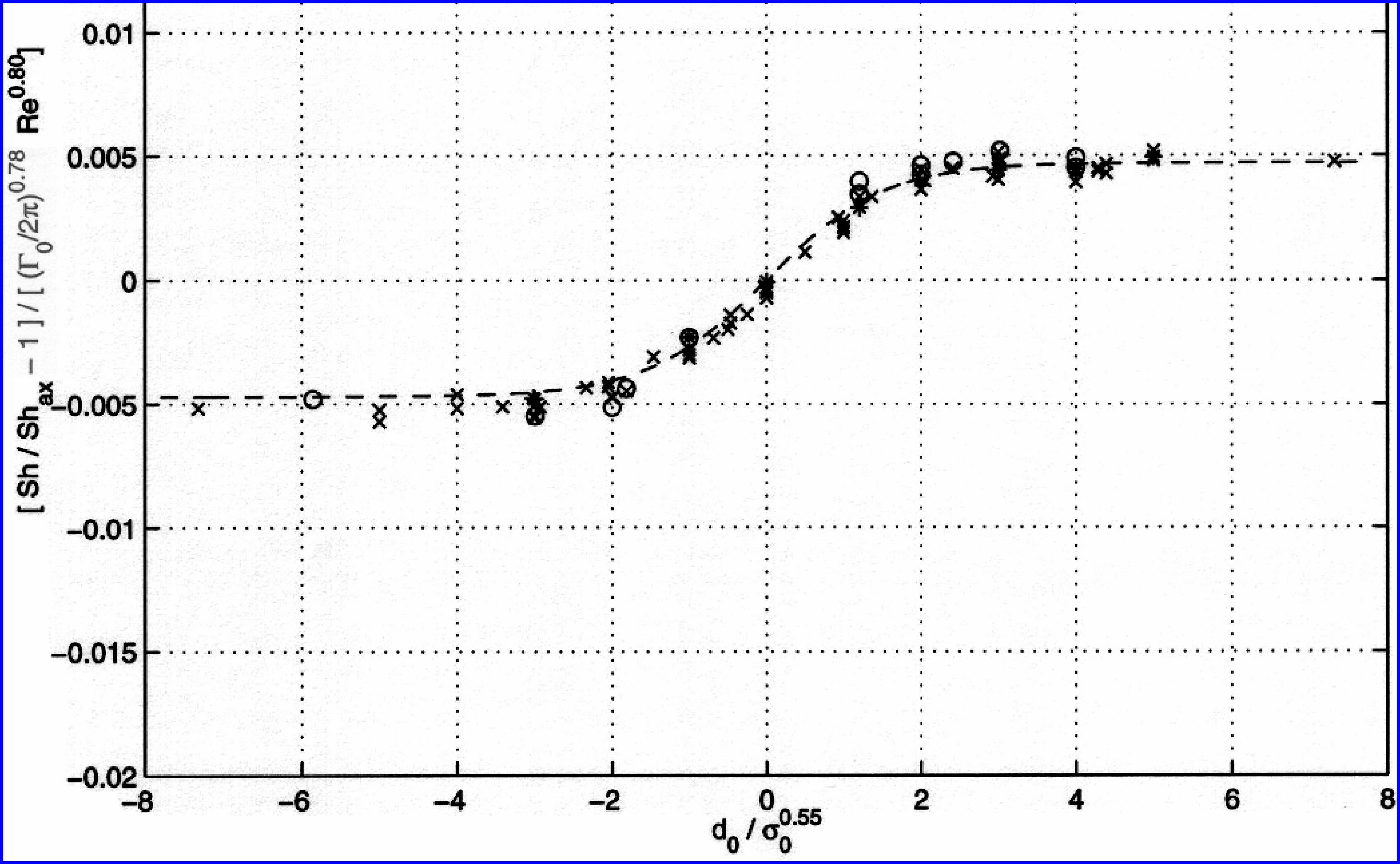
Significant deviations from an axisymmetric flow occur.

Time-Averaged Nusselt Number Modification with Liquid Sphere



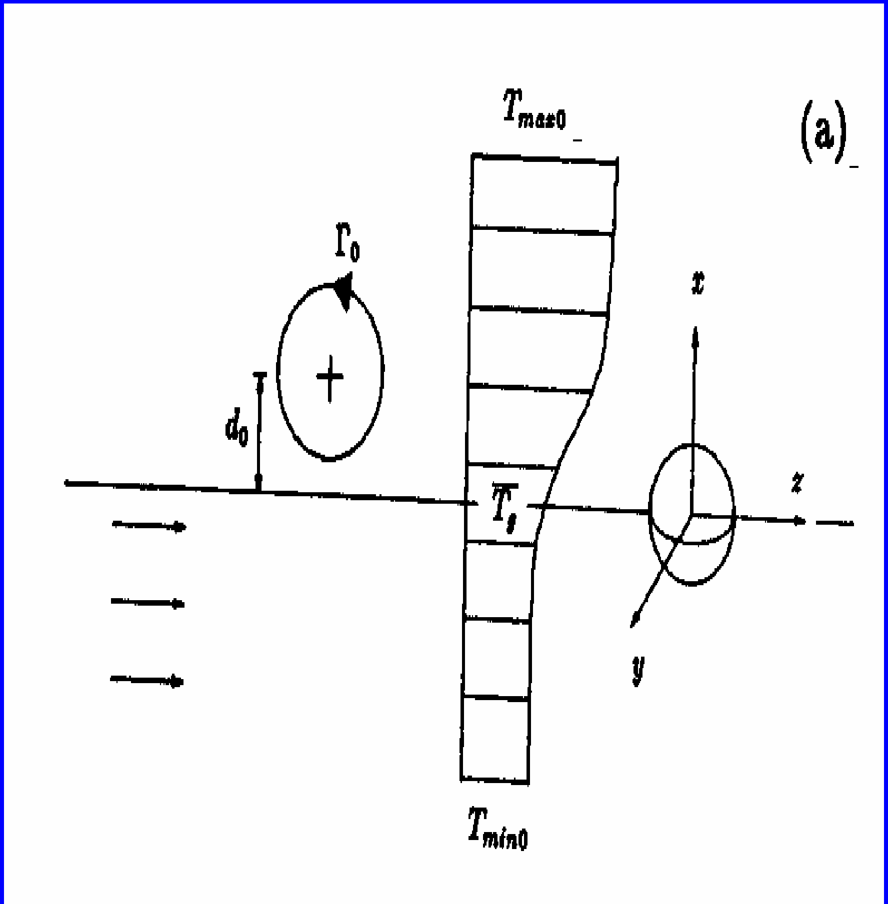
The hyperbolic tangent curve fits the data from N-S solutions very well

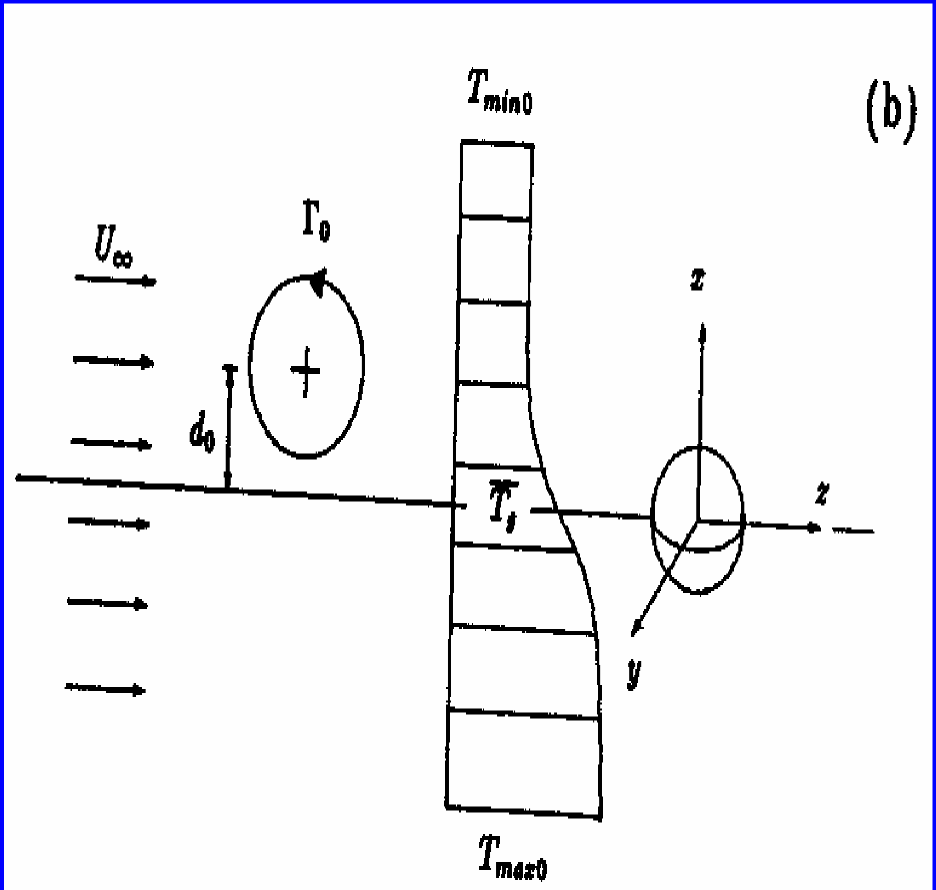
Sherwood Number Modification for a Vaporizing Droplet



The effect can be significant for large Re and/or Γ .

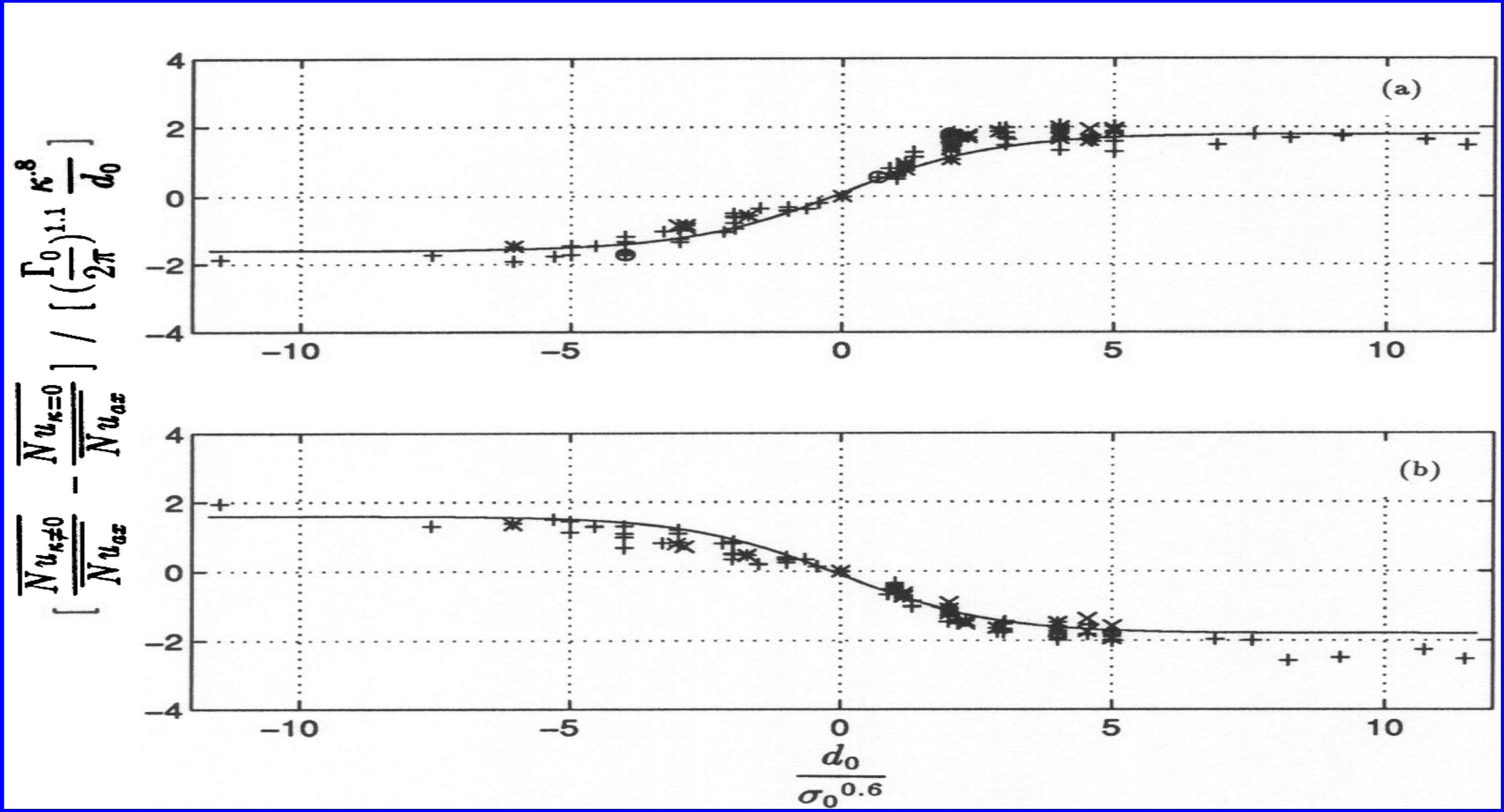
Effects of Gradients in Gas Flow





The mutual interactions amongst the droplet, vortex, and temperature gradient can cause dramatic variations in average Nusselt number.

Modification of Nusselt Number Due to Gradients



When gradients are present, the vortex has a more dramatic effect because the lateral motion brings hotter or colder fluid into contact with the sphere.

SUMMARY

- A new unified approach to laminar and turbulent spray flow computations has been outlined.
- New source terms and flux terms have been identified that relate to phenomena and gradients of the flow in the microstructure (sub-grid).
- The indications from certain model problems are that some of these terms can provide significant corrections to existing approaches. So, more research is needed.