Combustion Instability: Liquid-Propellant Rockets and Liquid-Fueled Ramjets

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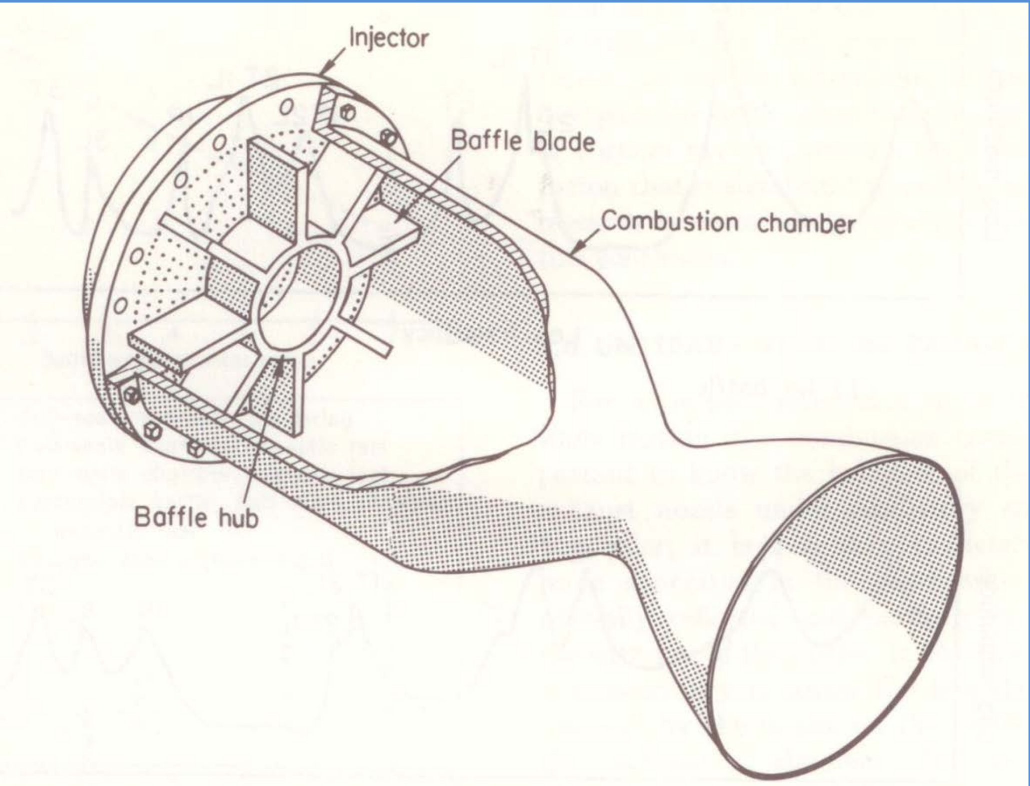
Linear Theory
Nonlinear Theory
Nozzle Admittance
Acoustic Liners
Vaporization Rate Control



It has been a very long time !

1965, Right to Left,
Professor Luigi Crocco
and his students,
Ben T. Zinn and
William A. Sirignano

3D Wave Equation

$$\frac{1}{\gamma}\left[\frac{\partial^2 p}{\partial t^2} - a^2\nabla^2 p\right] = a^2\frac{\partial M}{\partial t}$$

$$-a^2\nabla\cdot\left[\mathbf{F} + M\mathbf{V} - \nabla\cdot\rho\mathbf{V}\mathbf{V}\right]$$

$$+(\gamma-1)\,\frac{\partial}{\partial t}\left[G - \mathbf{V}\cdot\mathbf{F}\right]$$

$$-\frac{\partial}{\partial t}\left[p\mathbf{V}\cdot\nabla\sigma\right] + \frac{\partial p}{\partial t}\frac{\partial a^2}{\partial t}$$

Linearization and separation of variables in cylindrical combustion chamber

$$P = P^{(0)} + P^{(1)}$$

$$U = U^{(0)} + U^{(1)}$$

$$M = M^{(1)}$$

$$\Sigma = \Sigma^{(1)}$$

$$s = s^{(0)} + s^{(1)}$$

$$\psi_{\nu\eta}(r) = J_\nu(s_{\nu\eta} r) + BY_\nu(s_{\nu\eta} r)$$

$$\chi = s_{\nu\eta}$$

$$p' = P_{\nu\eta}(x)\psi_{\nu\eta}(r)e^{i\nu\theta} + \cdots$$

$$u' = U_{\nu\eta}(x)\psi_{\nu\eta}(r)e^{i\nu\theta} + \cdots$$

$$v' = V_{\nu\eta}(x)\,\frac{d\psi_{\nu\eta}}{dr}(r)e^{i\nu\theta} + \cdots$$

$$w' = W_{\nu\eta}(x)\,\frac{\psi_{\nu\eta}(r)}{r}\,e^{i\nu\theta} + \cdots$$

$$\sigma' = \Sigma_{\nu\eta}(x)\psi_{\nu\eta}(r)e^{i\nu\theta} + \cdots$$

$$M' = M_{\nu\eta}(x)\psi_{\nu\eta}(r)e^{i\nu\theta} + \cdots$$

$$I = I_{\nu\eta}(x)\psi_{\nu\eta}(r)e^{i\nu\theta} + \cdots$$

Wall Boundary Condition and Oscillation Mode

Full Cylinder

$$J_\nu'(s_{\nu\eta}^*) = 0$$

Annular Chamber

$$\frac{dJ_\nu}{dr}(s_{\nu\eta}^*)\frac{dY_\nu}{dr}(s_{\nu\eta}^*\xi) - \frac{dJ_\nu}{dr}(s_{\nu\eta}^*\xi)\frac{dY_\nu}{dr}(s_{\nu\eta}^*) = 0$$

$$B^* = -\frac{dJ_\nu}{dr}(s_{\nu\eta}^*\xi) \Big/ \frac{dY_\nu}{dr}(s_{\nu\eta}^*\xi)$$

Acoustically Lined Walls

$$s_{\nu\eta} \approx s_{\nu\eta}^* + \frac{\gamma s}{s_{\nu\eta}^*}\frac{y}{1-\left(\frac{\nu}{s_{\nu\eta}^*}\right)^2} \equiv \chi^* + \chi^{(1)}$$

ν	η	$s_{\nu\eta}$	Transverse character of mode
1	1	1.8413	First tangential
2	1	3.0543	Second tangential
0	2	3.8317	First radial
3	1	4.2012	Third tangential
0	3	7.0156	Second radial
1	2	5.3313	Combined first tangential and first radial
1	3	8.5263	Combined first tangential and second radial
2	2	6.7060	Combined second tangential and first radial

Modes of Transverse Oscillation

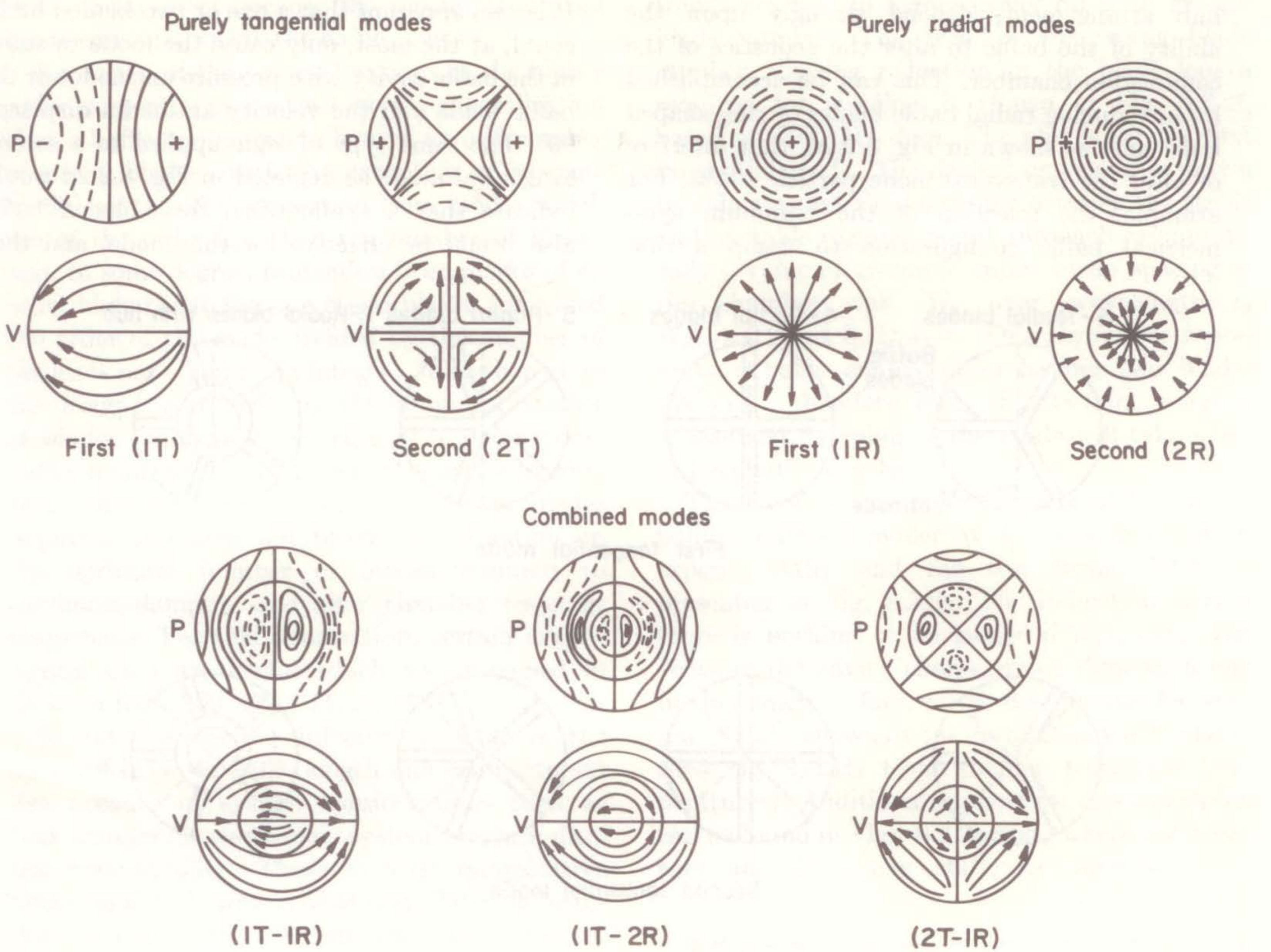


FIGURE 8.2.2b.—Transverse mode characteristics.

Radial and Tangential Velocities for Linear Perturbations

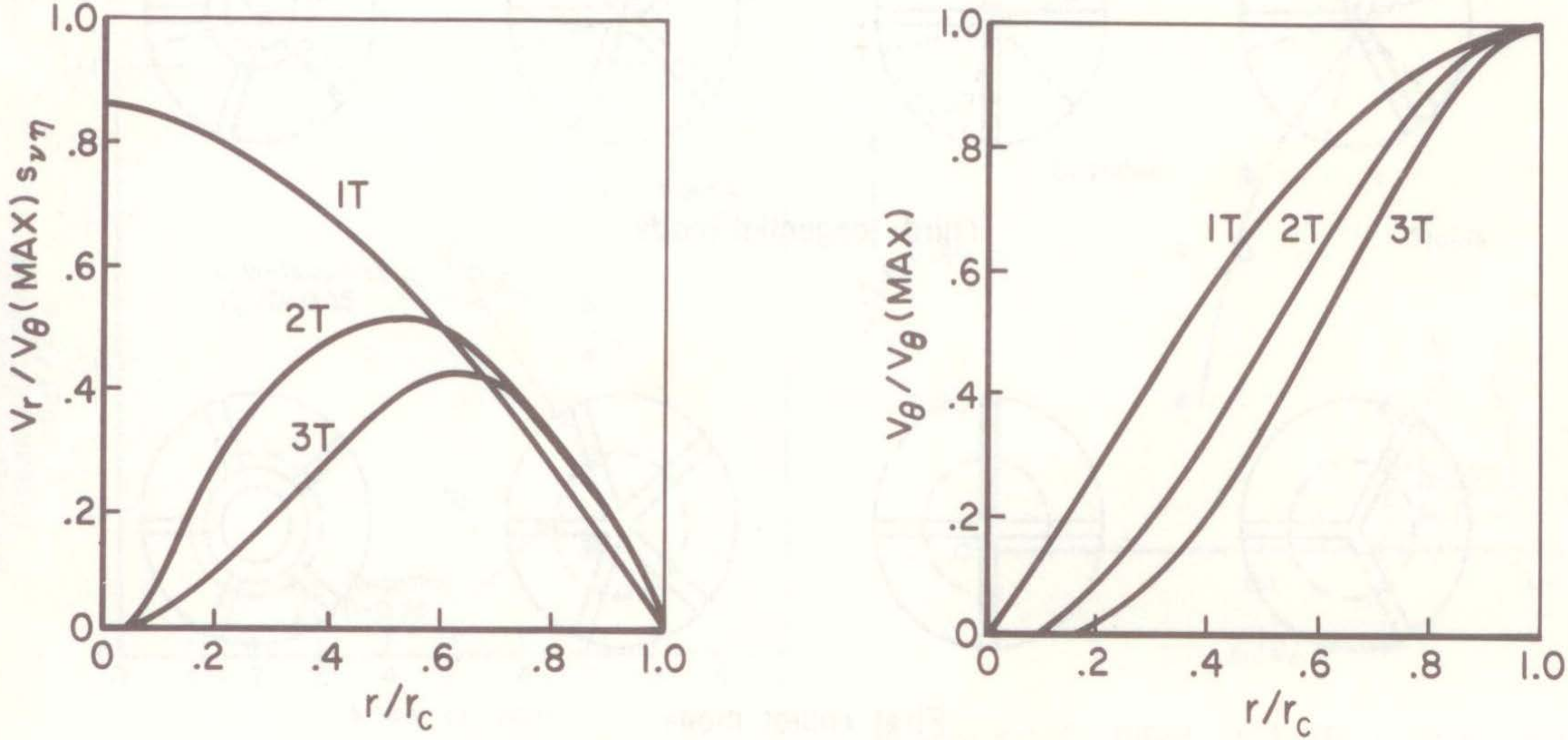


FIGURE 8.2.2c.—Tangential mode velocity profiles.

Determination of Frequency and Growth Rate

$$A\lambda^{(1)}x_e = \mathcal{E}_r - \frac{A\chi^*}{\omega^{(0)}} x_e \chi_i^{(1)} + \gamma \int_0^{x_e} M_r^{(1)} \cos \pi j \frac{x}{x_e}\, dx$$

$$- \frac{1}{2}\int_0^{x_e} k\bar{p}_L^{\circ} \left(1+ \cos 2\pi j \frac{x}{x_e}\right) dx$$

$$- (\gamma+1)\bar{u}_e + \left(\frac{j\pi}{\omega^{(0)}x_e}\right)^2 \bar{u}_e$$

$$- \frac{j\pi}{x_e}\left[\gamma - 2\left(\frac{\pi j}{x_e\omega^{(0)}}\right)^2\right]\int_0^{x_e} \bar{u}\sin 2j\pi \frac{x}{x_e}\, dx$$

Growth Rate

$$A\omega^{(1)}x_e = \mathcal{E}_i + \frac{A\chi^*}{\omega^{(0)}} x_e\chi_r^{(1)} + \gamma\int_0^{x_e} M_i^{(1)} \cos\pi j \frac{x}{x_e}\, dx$$

$$- \gamma\bar{u}_e - \omega^{(0)}(\gamma-2)\int_0^{x_e}\bar{u}\sin 2\omega^{(0)}x\,dx$$

Frequency

Crocco Sensitive-Time-lag Theory

$$t-\tau_T<t_1<t$$

$$\int_{t-\tau_T}^{t} \tilde{f}\, dt_1 = E_{\acute{a}}$$

$$n=\left(\frac{\partial \ln \tilde{f}}{\partial \ln p}\right) \qquad \tilde{f}=\bar{f}\left[1+n\,\frac{(p-\bar{p})}{\bar{p}}\right]$$

$$\int_{t-\tau_T}^{t-\tau} \bar{f}\, dt_1 + n\int_{t-\tau}^{t} \bar{f}\,\frac{(p-\bar{p})}{\bar{p}}\, dt_1 = \int_{t-\bar{\tau}_T}^{t} \bar{f}\, dt_1$$

$$\tau-\bar{\tau}=-n\int_{t-\bar{\tau}}^{t} \frac{p-\bar{p}}{\bar{p}}\, dt_1$$

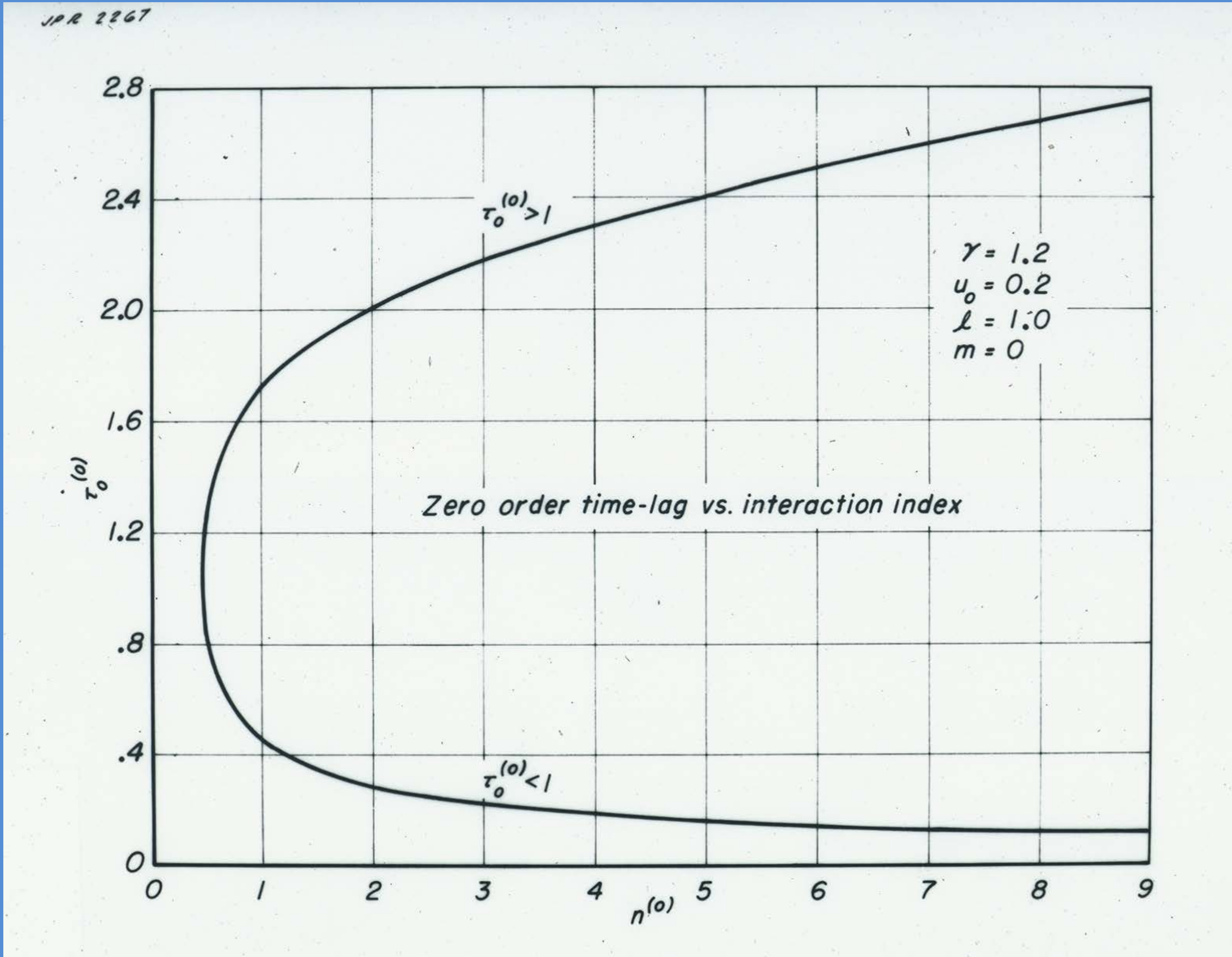
$$M\,dt=\bar{M}\,d(t-\tau)$$

$$M-\bar{M}=-\bar{M}\left(\frac{\partial \tau}{\partial t}\right)$$

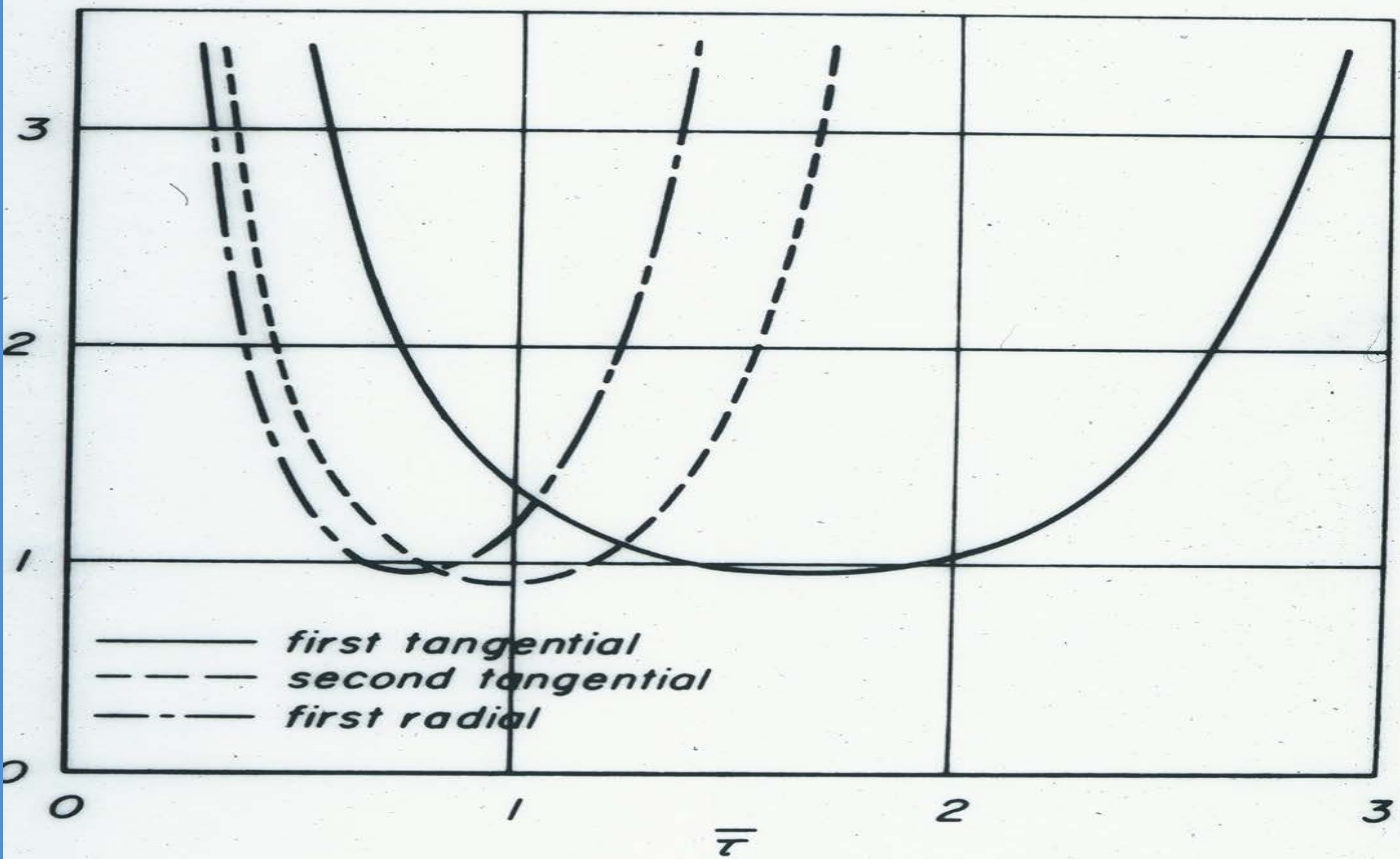
$$\frac{M-\bar{M}}{\bar{M}}=-\left(\frac{\partial \tau}{\partial t}\right)=+n\,\frac{p(t)-p(t-\bar{\tau})}{\bar{p}}$$

n and tau are two parameters dependent largely on propellant and injector design. The theory could be developed, alternatively, with a gain and phase, e.g., a complex number giving the ratio of burning-rate perturbation to pressure perturbation.

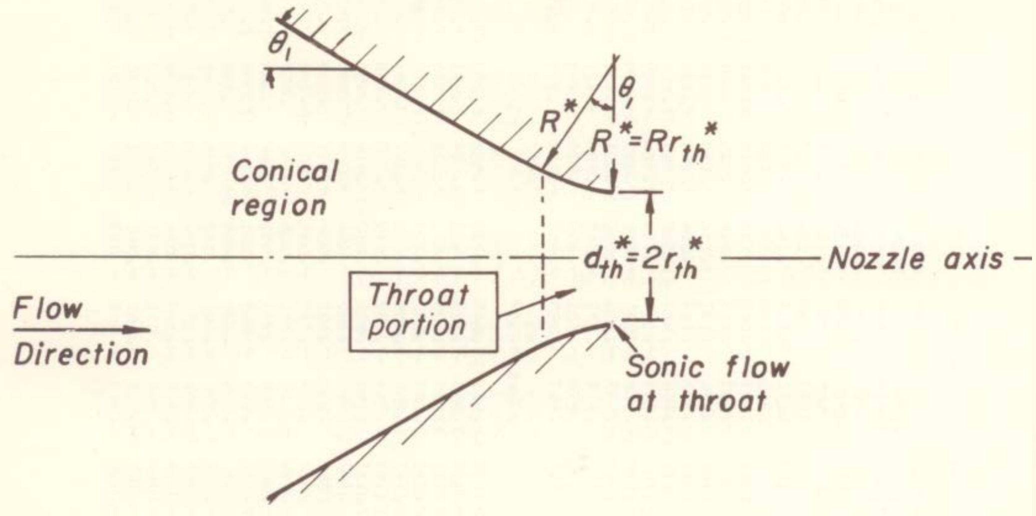
Stability Limits for Longitudinal Mode



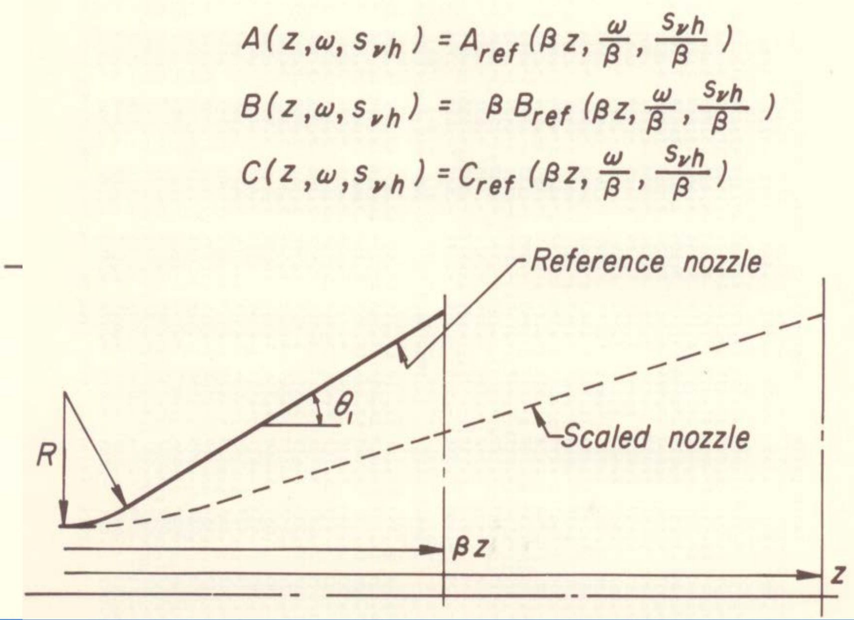
Theoretical stability limits for several purely transverse modes, uniformly distributed combustion, $\bar{u}_e = 0.10$



Nozzle Admittance Theory



1 Geometry of convergent portion of nozzle



-- Generally, the nozzle dimensions will be of the same order as the wavelength. Therefore, the determination of the waveform is important; the combustion chamber oscillation is implicitly affected through the use of an admittance function applied as a condition at the chamber exit (nozzle entrance).

-- Short (small) nozzles will behave in a quasi-steady manner.

-- Calculation results can be scaled to cover an infinite range of nozzle shapes.

Linear Nozzle Flow Oscillations

$$\frac{\rho'}{\rho} = R(\phi)\Psi(\psi)\Theta(\theta)e^{i\omega t}$$

$$\frac{u'}{\bar{q}} = U(\phi)\Psi(\psi)\Theta(\theta)e^{i\omega t}$$

$$\frac{v'}{r\bar{\rho}\,\bar{q}} = V(\phi)\Psi'(\psi)\Theta(\theta)e^{i\omega t}$$

$$rw' = W(\phi)\Psi(\psi)\Theta'(\theta)e^{i\omega t}$$

$$\frac{p'}{\gamma\bar{\rho}} = P(\phi)\Psi(\psi)\Theta(\theta)e^{i\omega t}$$

$$\omega = \frac{L^\star}{\bar{c}^{o\star}}\omega^\star .$$

$$u' = \mathcal{U}(z)J_\nu(s_{\nu h}r)\Theta(\theta)e^{i\omega t}$$

$$v' = \mathcal{V}(z)\frac{dJ_\nu}{dr}(s_{\nu h}r)\Theta(\theta)e^{i\omega t}$$

$$p' = \mathcal{P}(z)J_\nu(s_{\nu h}r)\Theta(\theta)e^{i\omega t}$$

$$s' = \mathcal{S}(z)J_\nu(s_{\nu h}r)\Phi(\theta)e^{i\omega t} ,$$

Analysis of Longitudinal Variation Solution at Singular Point (Nozzle Throat)

$$U = \Phi'$$

$$f_0(\phi) = e^{-i\omega \int_0^{\phi} \frac{d\phi'}{\overline{q^2}(\phi')}},$$

$$f_1(\phi) = \frac{1}{2}\int_{\phi_{th}}^{\phi} f_0(\phi')\frac{d\overline{q^2}}{d\phi'}d\phi'$$

$$f_2(\phi) = f_0(\phi)\int_{\phi_{th}}^{\phi} \frac{f_1(\phi')}{\overline{q^2}(\phi')f_0(\phi')}d\phi' .$$

$$\overline{q^2}(\overline{c^2} - \overline{q^2})\Phi'' - \overline{q^2}\left(\frac{1}{\overline{c^2}}\frac{d\overline{q^2}}{d\phi} + 2i\omega\right)\Phi' + \left(\omega^2 - \frac{\gamma - 1}{2}\frac{i\omega\overline{q^2}}{\overline{c^2}}\frac{d\overline{q^2}}{d\phi} - \frac{s_{\nu h}^2}{2\psi_w}\bar{\rho}\bar{q}\overline{c^2}\right)\Phi$$

$$= -\sigma\overline{c^2}\left[i\omega\left(\frac{f_1}{\overline{c^2}} - f_0\right) + \overline{q^2}\frac{d}{d\phi}\left(\frac{f_1}{\overline{c^2}} - f_0\right) + \frac{s_{\nu h}^2}{2\psi_w}\bar{\rho}\bar{q}f_2\right] - C_1\frac{s_{\nu h}^2}{2\psi_w}\bar{\rho}\bar{q}\overline{c^2}f_0 .$$

$$U + AP + BV + CS = 0 ,$$

Special Case of Isentropic Flow

A first-order ordinary differential equation can be formed; it is integrated starting from the throat in the upstream direction, using the analytic solution at the throat. It directly Yields information about the admittance function.

$$\frac{d}{d\phi}[(\overline{c^2} - \overline{q^2}]\xi^{(j)}] + \left(\zeta - \frac{i\omega}{\overline{q^2}} - \frac{2i\omega}{\overline{c^2} - \overline{q^2}}\right)[(\overline{c^2} - \overline{q^2})\xi^{(j)}] = \frac{1}{\overline{q^2}}F^{(j)}$$

$$\zeta(\phi) = \frac{1}{\Phi_h}\frac{d\Phi_h}{d\phi} \; ; \; \Phi_h = \exp\left\{\int^{\phi}\zeta(\phi')\,d\phi\right\}$$

$$\xi^{(j)}(\phi) = \frac{1}{\overline{c^2}f_0\Phi_h}\left[\Phi^{(j)}\frac{d\Phi_h}{d\phi} - \Phi_h\frac{d\Phi^{(j)}}{d\phi}\right]$$

$$U = C_2\frac{d\Phi_h}{d\phi}, \qquad V = C_2\Phi_h, \qquad P + \overline{q^2}U = -C_2 i\omega\Phi_h$$

$$U = \zeta V = -\frac{\zeta}{\overline{q^2}\zeta + i\omega}P = \frac{\alpha}{\gamma}P$$

Similarly, the general case can be recast with first-order ODEs integrated starting from the throat.

$$C = \left(\frac{2}{\gamma + 1}\right)^{\frac{1}{2}} \overline{q}\,\overline{c}^{2} \, \frac{\frac{1}{2}(1 - \overline{q^2})\hat{\xi}^{(1)} - \frac{1}{2} i\hat{\omega}\xi^{(2)} + f_3\hat{\zeta}}{\overline{q^2}(\overline{c^2}\hat{\xi}^{(1)} - \hat{\zeta}) - \frac{1}{2} i\hat{\omega}} \Bigg| .$$

$$\left(\frac{d\hat{\zeta}}{d\hat{\phi}} + \hat{\zeta}^2\right) = (g + ih)\hat{\zeta} - (j - ik)$$

$$\frac{d}{d\hat{\phi}}\left[(1 - \overline{q^2})\hat{\xi}^{(1)}\right] = -\left\{\hat{\zeta} - i\hat{\omega}\left(\frac{1}{2\overline{q^2}} + \frac{2}{(\gamma + 1)(1 - \overline{q^2})}\right)\right\}\left[(1 - \overline{q^2})\hat{\xi}^{(1)}\right] + \frac{2}{\gamma + 1}\,\frac{\hat{s}_{\nu h}^2}{4}\,\frac{\overline{c}^{2/(\gamma-1)}}{\overline{q}}$$

$$\frac{d}{d\hat{\phi}}\left[(1 - \overline{q^2})\xi^{(2)}\right] = -\left\{\hat{\zeta} - i\hat{\omega}\left(\frac{1}{2\overline{q^2}} + \frac{2}{(\gamma + 1)(1 - \overline{q^2})}\right)\right\}\left[(1 - \overline{q^2})\xi^{(2)}\right] + \frac{2}{\gamma + 1}\left\{\frac{df_3}{d\hat{\phi}} + \frac{\hat{s}_{\nu h}^2\,\overline{c}^{2/(\gamma-1)}\,\overline{q}}{2i\hat{\omega}}\left(\frac{1 - \overline{q^2}}{2\overline{q^2}} + \frac{\overline{c^2}}{\overline{q^2}} f_3\right)\right\}$$

A certain combination of admittance coefficients is useful for transverse oscillations.

$$\alpha = -\left(\frac{\gamma + 1}{2}\right)^{(\gamma+1)/\{2(\gamma-1)\}} \frac{\bar{q}}{\bar{c}^{2/(\gamma-1)}} \frac{\hat{\zeta}}{\bar{q}^2\hat{\zeta} + i\hat{\omega}/2}$$

$$\mathcal{E} = \gamma\mathcal{A} - \frac{\mathcal{B}}{i\omega_c},$$

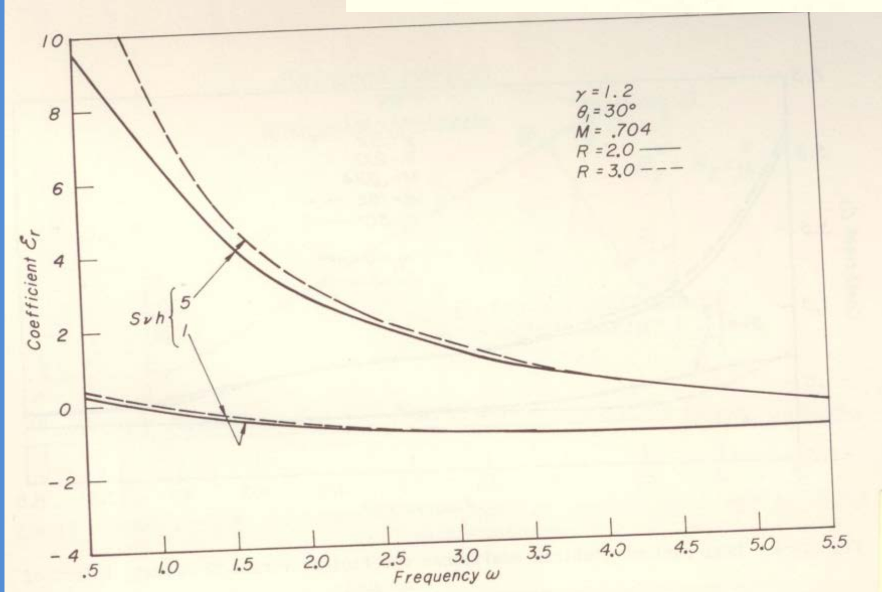


Fig. 16(a) Real part of combined admittance coefficient versus frequency: Effect of throat wall curvature

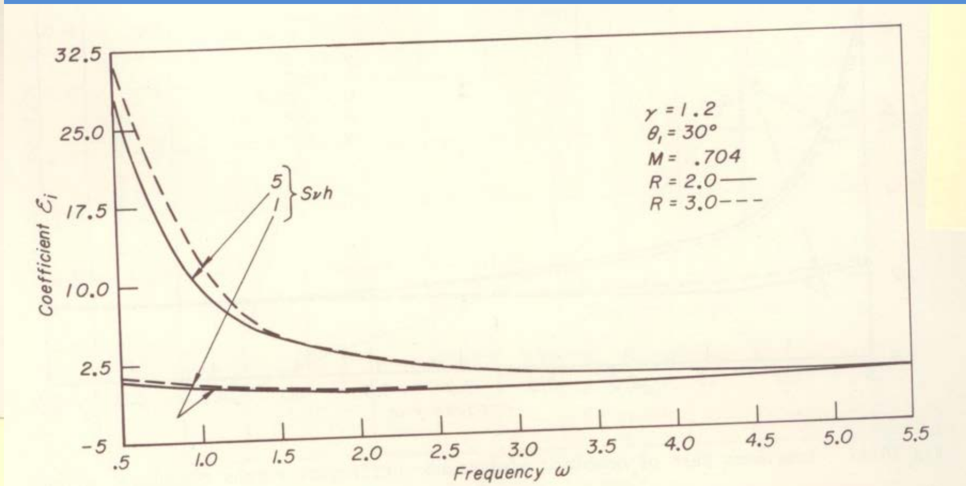
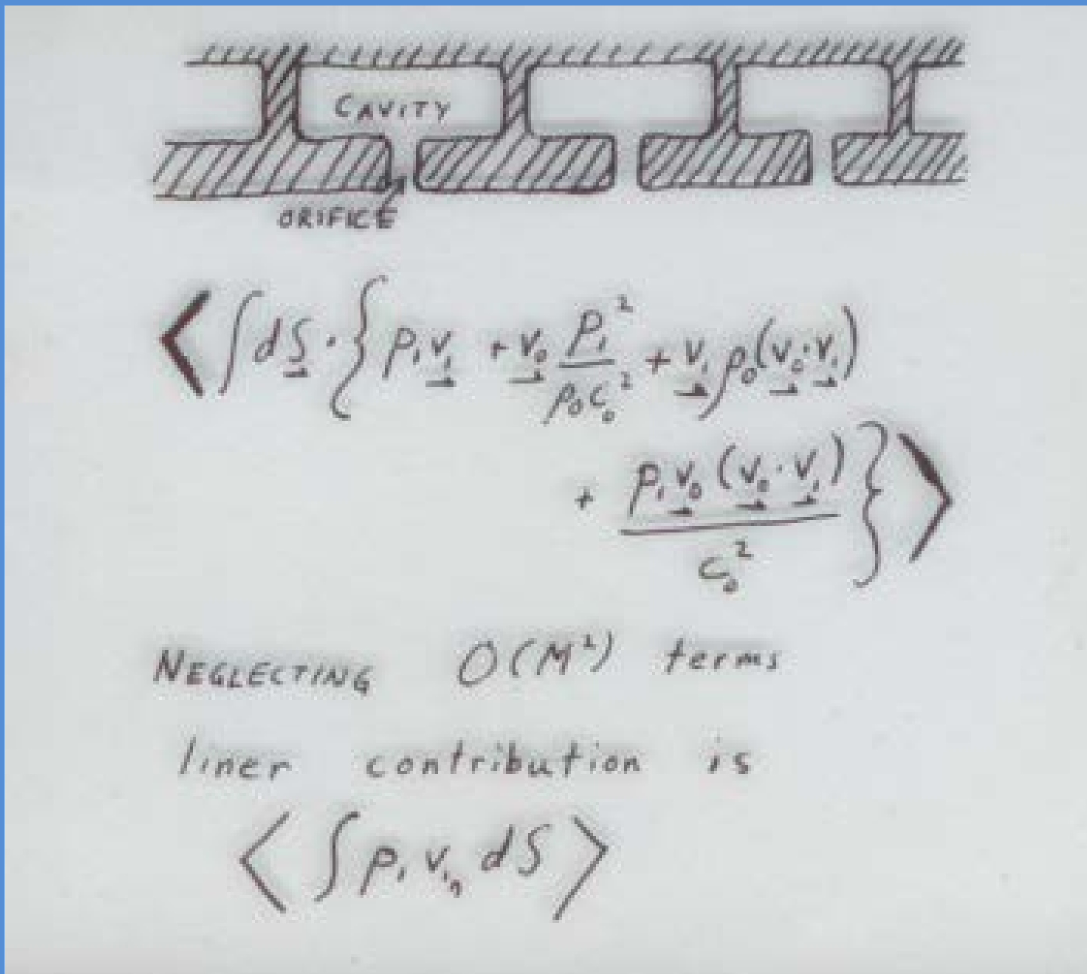


Fig. 16(b) Imaginary part of combined admittance coefficient versus frequency: Effect of throat wall curvature

Acoustic Liners



Acoustic Liners can be Helmholtz resonators, quarter wave tubes, or a hybrid of both.

They take energy from the outer flow. The primary nonlinear dissipative mechanism is the exit jet which alternates position at each end of the orifice.

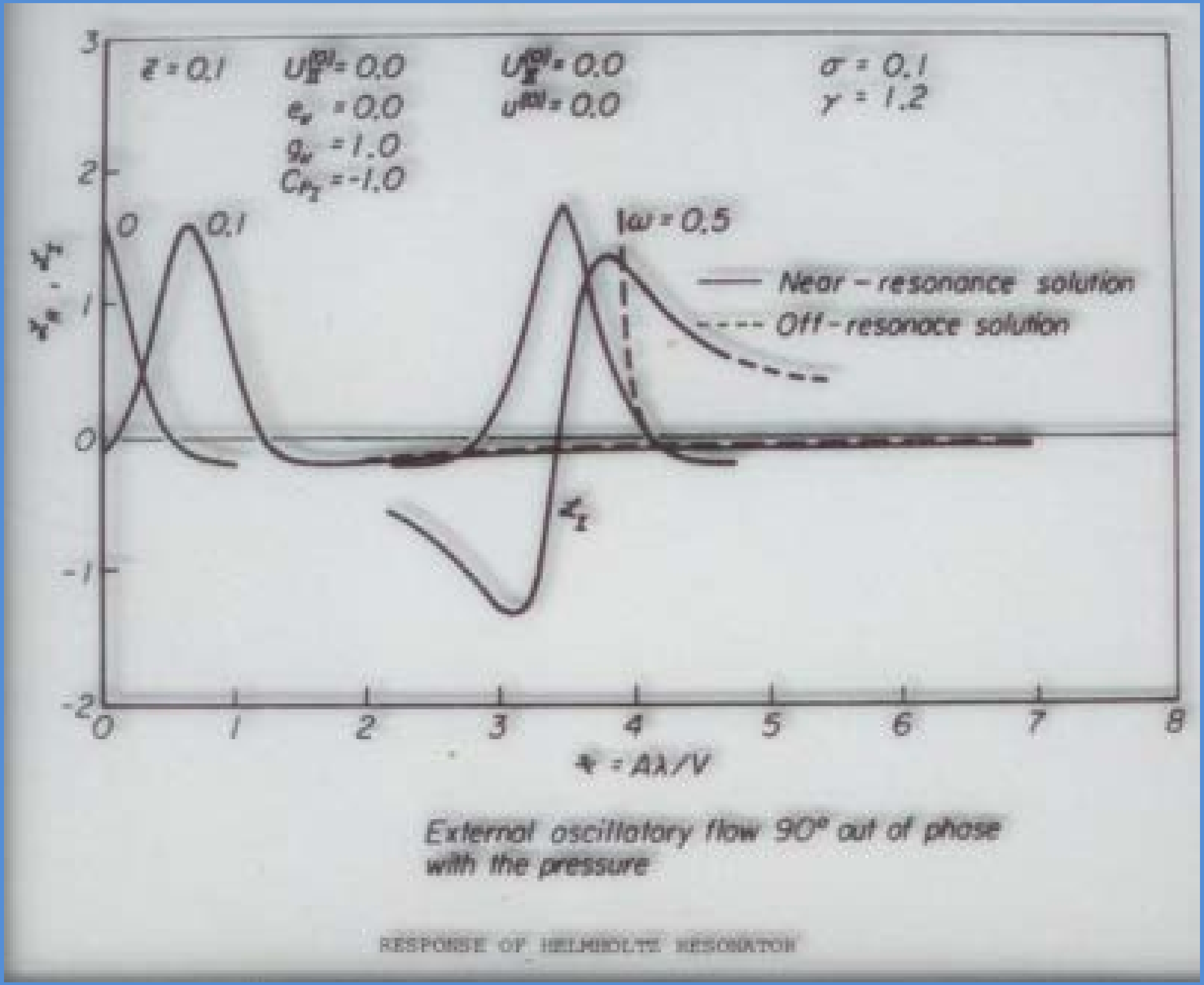
Acoustic Liner Perturbations

The physics of the Helmholtz resonator involve four simple aspects:

1) **an oscillating external pressure which drives the Helmholtz resonator oscillation;**
2) **a cavity which essentially is a capacitor with time-varying mass and density due to alternating mass influx and efflux ;**
3) **oscillatory motion in the orifice with friction losses that are the primary linear dissipative mechanism; and**
4) **the jet that alternates position between the two orifice ends and provides the stagnation pressure loss which is the primary nonlinear dissipative mechanism.**

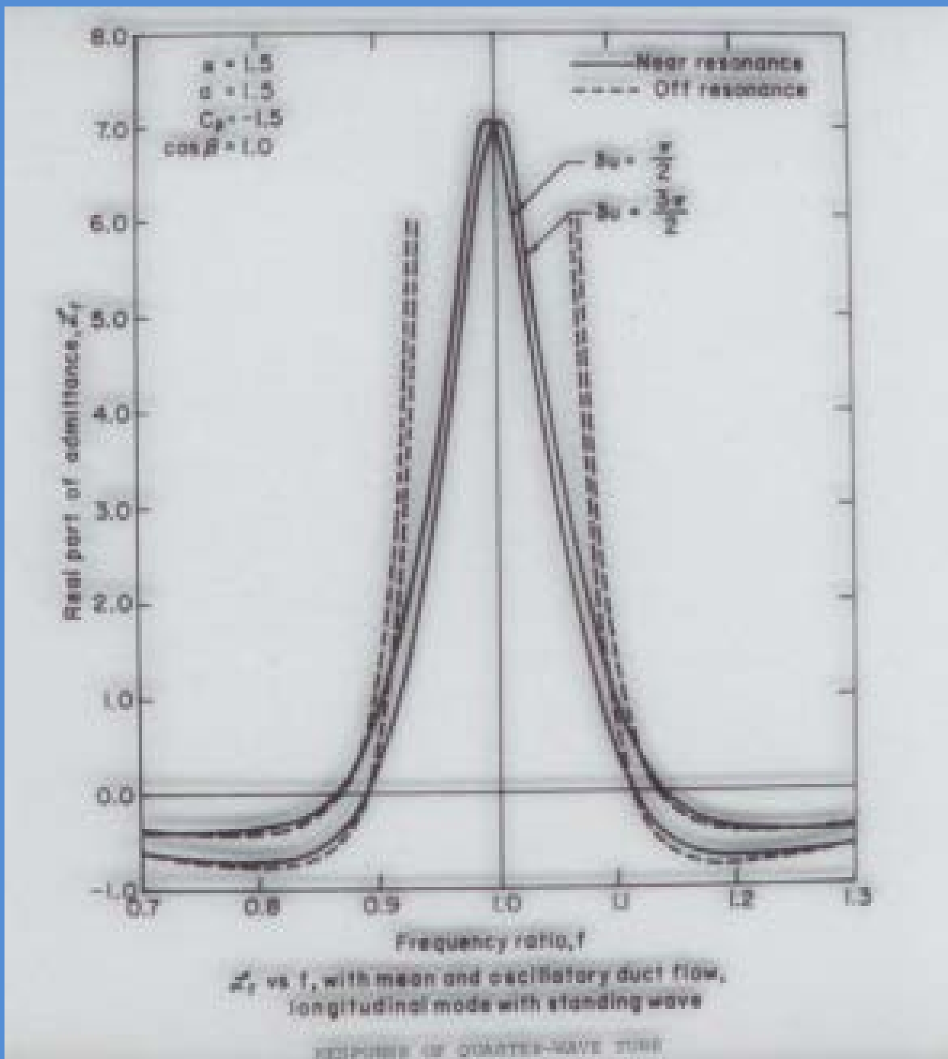
The pressure drop across the jet is proportional to the square of the jet velocity as dictated by Bernoulli's Law. Therefore, in the perturbation scheme here, pressure perturbation is made proportional to the amplitude parameter while velocity perturbation is proportioned to the square root of that amplitude parameter.

Admittance for Helmholtz Resonator

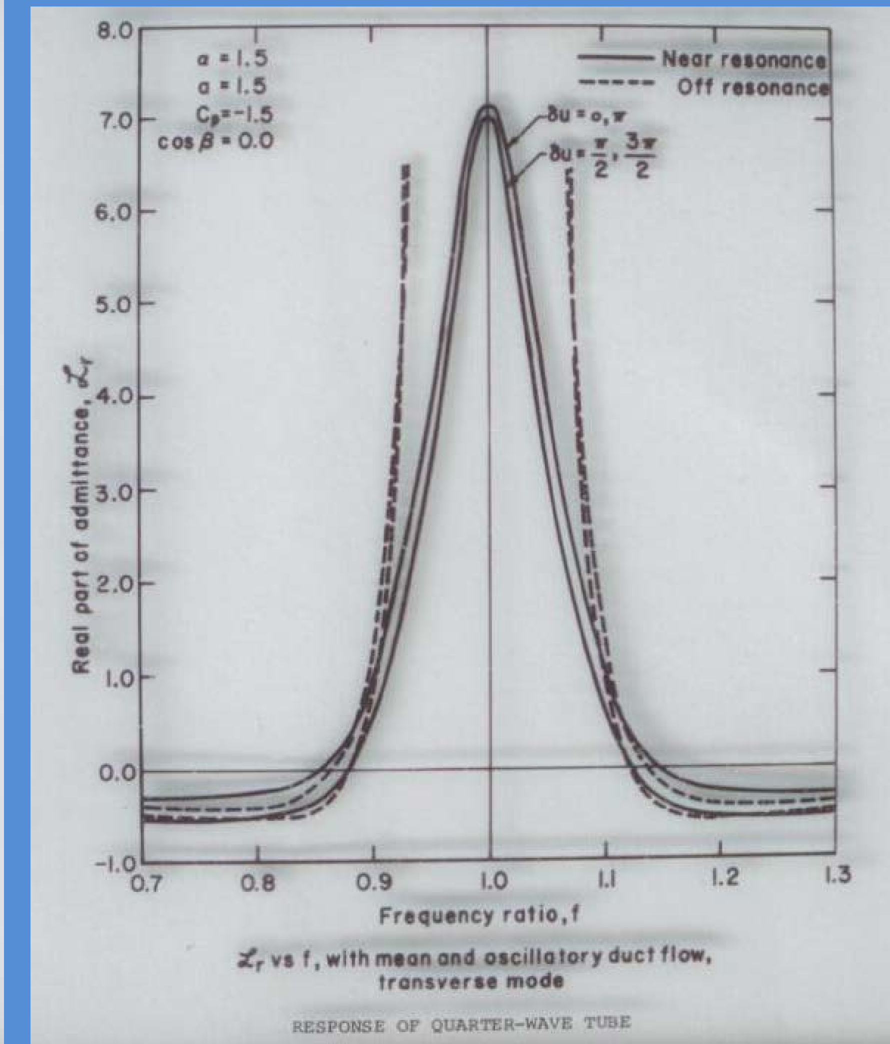


Admittance for Quarter-Wave Tube

Longitudinal standing wave



Transverse travelling wave



Nonlinear Longitudinal Mode Instability with Shocks

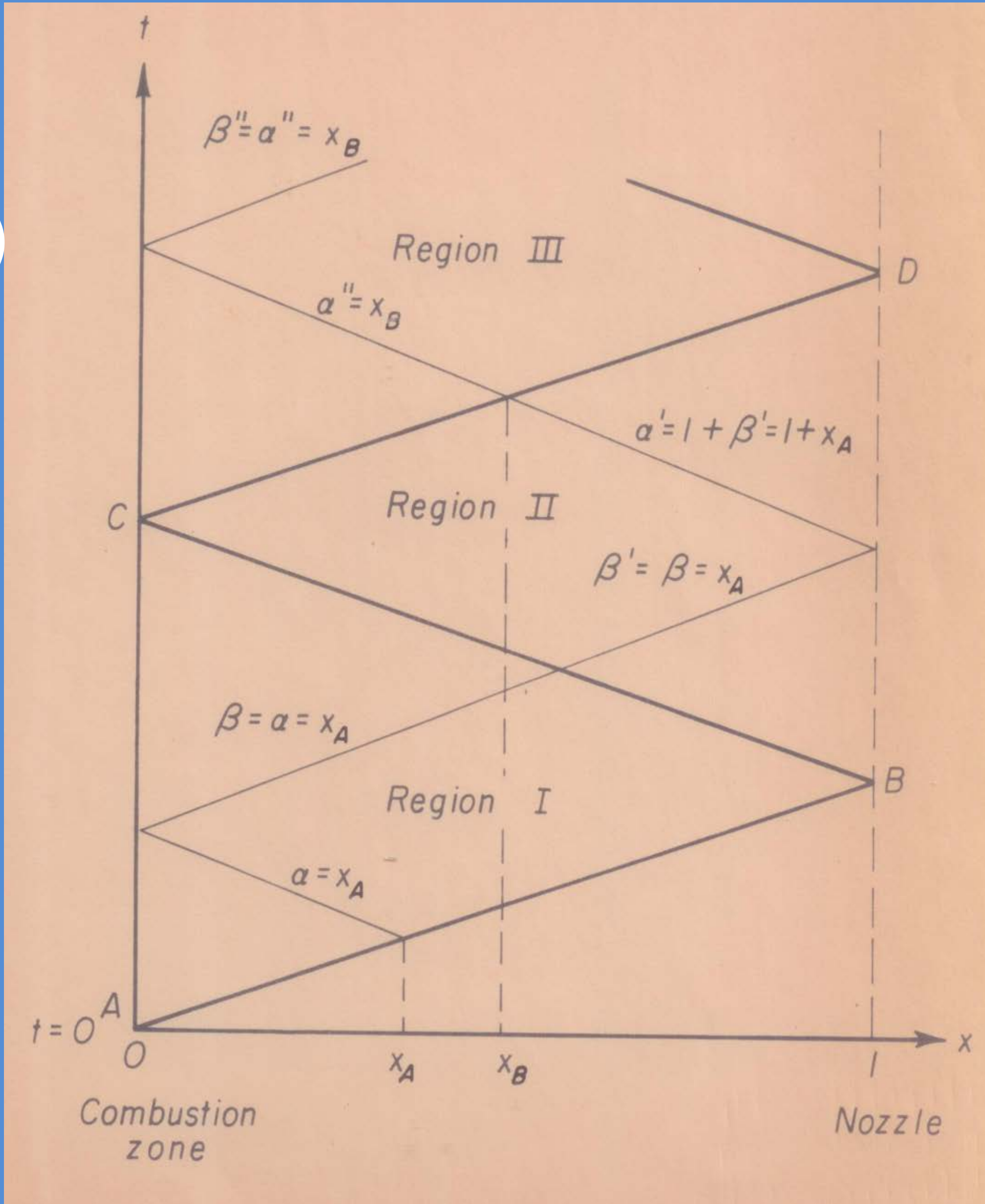
Premixed Combustion (no time lag)

Concentrated Combustion Zone

Short (Quasi-steady) Nozzle

Use of Characteristic Coordinates (method of strained coordinates, PLK method)

Limit Cycle



Perturbation Method

$$u = u_0 + \varepsilon u_1(\alpha,\beta) + \varepsilon^2 u_2(\alpha,\beta) + ----$$

$$a = 1 + \varepsilon a_1(\alpha,\beta) + \varepsilon^2 a_2(\alpha,\beta) + ----$$

$$p = 1 + \varepsilon p_1(\alpha,\beta) + \varepsilon^2 p_2(\alpha,\beta) + ----$$

$$x = x_0(\alpha,\beta) + \varepsilon x_1(\alpha,\beta) + \varepsilon^2 x_2(\alpha,\beta) + ----$$

$$t = t_0(\alpha,\beta) + \varepsilon t_1(\alpha,\beta) + \varepsilon^2 t_2(\alpha,\beta) + ----$$

$$n = n_0 + \epsilon n_1 + \epsilon^2 n_2 + - -$$

$$\omega = \omega_0 + \epsilon \omega_1 + \epsilon^2 \omega_2 + - -$$

$$T = T_0 + \epsilon T_1 + \epsilon^2 T_2 + - - -$$

$$\tau = \tau_0 + \epsilon \tau_1 + \epsilon^2 \tau_2 + - - - -$$

ε is the perturbation parameter and can be considered as a measure of the amplitude of the oscillation. Coordinates are perturbed here as well – An extension of Poincare’s method.

One-dimensional, Unsteady Gasdynamic Equations

$$\frac{\partial \psi_0}{\partial \alpha} = (u_0 + 1)\frac{\partial t_0}{\partial \alpha} \; ; \; \frac{\partial \psi_0}{\partial \beta} = (u_0 - 1)\frac{\partial t_0}{\partial \beta} \qquad (3)$$

$$\frac{2}{\gamma-1}\frac{\partial a_1}{\partial \alpha} + \frac{\partial u_1}{\partial \alpha} = 0 \; ; \; \frac{2}{\gamma-1}\frac{\partial a_1}{\partial \beta} - \frac{\partial u_1}{\partial \beta} = 0 \qquad (4)$$

$$\frac{\partial \psi_1}{\partial \alpha} = (u_0 + 1)\frac{\partial t_1}{\partial \alpha} + (u_1 + a_1)\frac{\partial t_0}{\partial \alpha}$$
$$\frac{\partial \psi_1}{\partial \beta} = (u_0 - 1)\frac{\partial t_1}{\partial \beta} + (u_1 - a_1)\frac{\partial t_0}{\partial \beta} \qquad (5)$$

$$\frac{2}{\gamma-1}\frac{\partial a_2}{\partial \alpha} + \frac{\partial u_2}{\partial \alpha} = 0 \; ; \; \frac{2}{\gamma-1}\frac{\partial a_2}{\partial \beta} - \frac{\partial u_2}{\partial \beta} = 0 \qquad (6)$$

Constant properties

Inviscid and isentropic except through shocks

Riemann invariants are used

$$\frac{2}{\gamma-1} a_i + u_i = P_i(\beta)$$
$$\frac{2}{\gamma-1} a_i - u_i = Q_i(\alpha)$$

so that

$$u_i = \frac{P_i(\beta) - Q_i(\alpha)}{2}$$

$$a_i = \frac{\gamma-1}{4}\left[P_i(\beta) + Q_i(\alpha)\right]$$

Boundary Conditions

Combustion Zone

$$u = u_0 + \omega u_0 \, (a-1) + \delta u_0 \, (a-1)^2 + O(a-1)^3 \quad (9)$$

$$u_1 = \omega u_0 \, a_1$$

$$\varepsilon = \nu \, (\omega - 1)$$

Entrance to Short Nozzle

$$u_i = u_0 \, a_i \quad \text{at} \quad \alpha' = 1 + \beta'$$

$$Q_i' = \frac{1-\nu}{1+\nu} \, P_i' \quad \text{at} \quad \alpha' = 1 + \beta'$$

$$\nu \equiv \frac{\gamma - 1}{2} \, u_0$$

The amplitude parameter ε and the curvature parameter λ are determined from the second-order analysis (similar to Poincare's elimination of secular terms)

$$r = \frac{16}{\gamma+1} \; \frac{1}{(1-v^2)\left[\frac{1-u_0}{1+u_0}\,\frac{1+v}{1-v} + \frac{1+u_0}{1-u_0}\right]}$$

$$\varepsilon = u_0\left[(\gamma-1)\frac{E}{RT^*} - \frac{3\gamma-1}{2}\right]$$

$$\lambda = \frac{16u_0\left(\frac{\gamma-1}{2}\,\frac{1}{1-v}\right)^2\left[2\left(\frac{E}{RT^*}\right)^2 - \frac{7\gamma-3}{\gamma-1}\,\frac{E}{RT^*} + \frac{\gamma(3\gamma-1)}{(\gamma-1)^2}\right]}{(\gamma+1)(1+v)\left[\frac{1-u_0}{1+u_0}\,\frac{1+v}{1-v} + \frac{1+u_0}{1-u_0}\right]}$$

$\gamma = 1.2$ and $u_o = .1$

$\frac{E}{RT^*}$	ε	λ	r	ΔP_{shock}
10.0	.07	.028	3.53	.152
12.5	.12	.189	3.53	.259
15.0	.17	.439	3.53	.362
17.5	.22	.778	3.53	.453
20.0	.27	1.207	3.53	.522

Flowfield Solutions

Region I

$$u-u_0 = \varepsilon \frac{r}{\lambda}\left\{\frac{e^{-\lambda\left(\frac{1-u_0^2}{2}\right)t}}{1+e^{-\lambda}}\left[\frac{1+\nu}{1-\nu}\, e^{\frac{\lambda}{2}(1-u_0)\tau} - e^{-\frac{\lambda}{2}(1+u_0)\tau}\right] - \frac{\nu}{1-\nu}\right\} + O(\varepsilon^2)$$

$$P-P_0 = \varepsilon\gamma \frac{r}{\lambda}\left\{\frac{e^{-\lambda\left(\frac{1-u_0^2}{2}\right)t}}{1+e^{-\lambda}}\left[\frac{1+\nu}{1-\nu}\, e^{\frac{\lambda}{2}(1-u_0)\tau} + e^{-\frac{\lambda}{2}(1+u_0)\tau}\right] - \frac{1}{1-\nu}\right\} + O(\varepsilon^2)$$

Region II

$$u-u_0 = \varepsilon \frac{r}{\lambda}\left\{\frac{e^{-\lambda\left(\frac{1-u_0^2}{2}\right)t}}{1+e^{-\lambda}}\left[\frac{1+\nu}{1-\nu}\, e^{\frac{\lambda}{2}(1-u_0)\tau} - e^{-\frac{\lambda}{2}(1+u_0)\tau}\, e^{\lambda}\right] - \frac{\nu}{1-\nu}\right\} + O(\varepsilon^2)$$

$$P-P_0 = \varepsilon\gamma \frac{r}{\lambda}\left\{\frac{e^{-\lambda\left(\frac{1-u_0^2}{2}\right)t}}{1+e^{-\lambda}}\left[\frac{1+\nu}{1-\nu}\, e^{\frac{\lambda}{2}(1-u_0)\tau} + e^{-\frac{\lambda}{2}(1+u_0)\tau}\, e^{\lambda}\right] - \frac{1}{1-\nu}\right\} + O(\varepsilon^2)$$

Shocks

$$\Delta u_{AB} = \varepsilon \frac{r}{\lambda}\,\frac{1+\nu}{1-\nu}\,\frac{1-e^{-\lambda}}{1+e^{-\lambda}} + O(\varepsilon^2)$$

$$\Delta u_{BC} = \varepsilon \frac{r}{\lambda}\,\frac{1-e^{-\lambda}}{1+e^{-\lambda}} + O(\varepsilon^2)$$

$$\Delta P_{AB} = \varepsilon\gamma \frac{r}{\lambda}\,\frac{1+\nu}{1-\nu}\,\frac{1-e^{-\lambda}}{1+e^{-\lambda}} + O(\varepsilon^2)$$

$$\Delta P_{BC} = \varepsilon\gamma \frac{r}{\lambda}\,\frac{1-e^{-\lambda}}{1+e^{-\lambda}} + O(\varepsilon^2)$$

Also, the shock velocities are

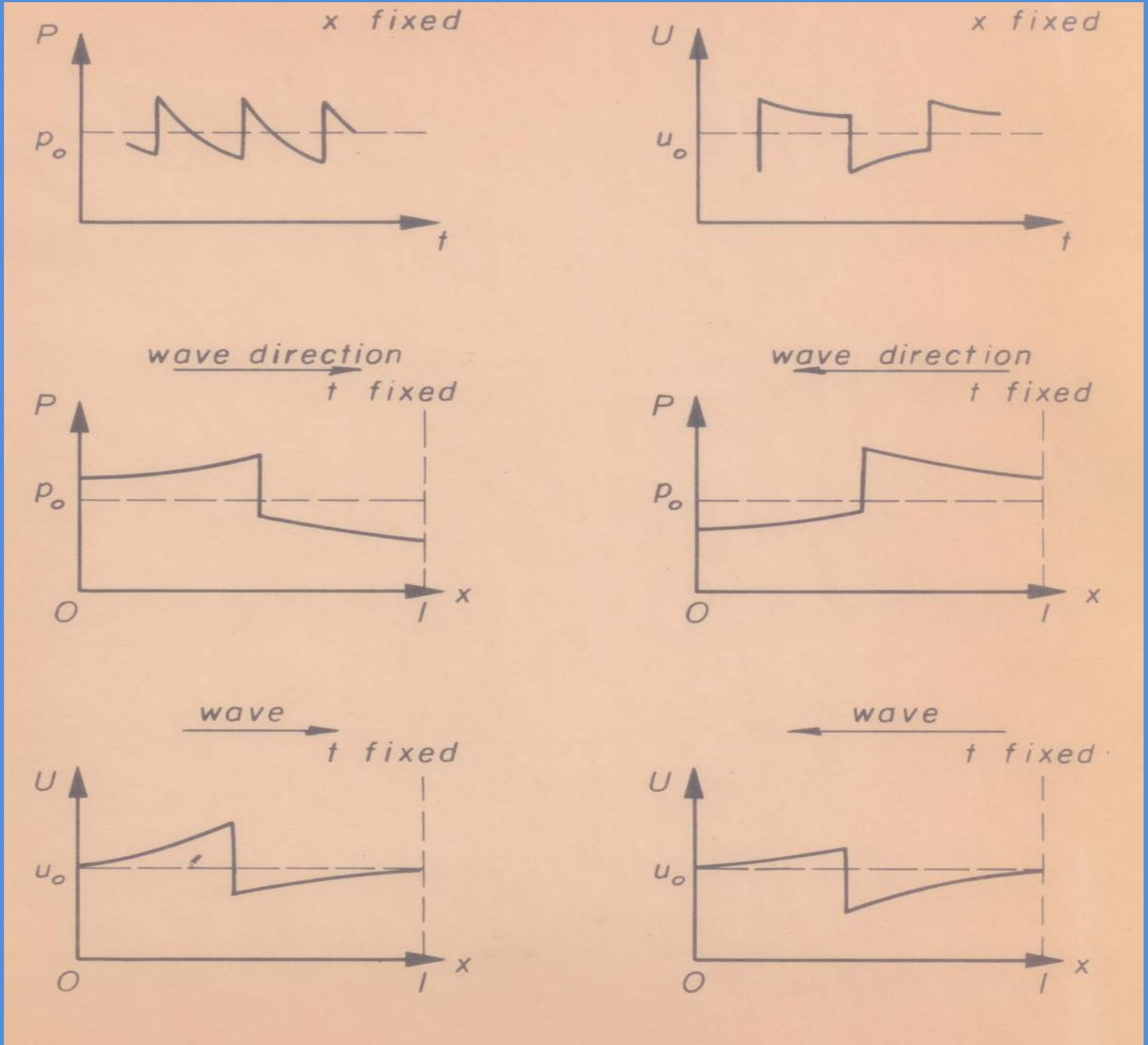
$$V_{AB} = 1+u_0 + \varepsilon \frac{r}{\lambda}\left\{\frac{3-\gamma}{4} - \frac{3-\gamma}{2}\,\frac{e^{-\lambda(1+u_0)t}}{1+e^{-\lambda}}\right\} + O(\varepsilon^2)$$

$$V_{BC} = u_0 - 1 + \varepsilon \frac{r}{\lambda}\,\frac{1+\nu}{1-\nu}\left\{\frac{\gamma-3}{4} + \frac{3-\gamma}{2}\,\underline{e^{\lambda\left(\frac{1}{1+u_0}-t\right)(1-u_0)}}\right\} + O(\varepsilon^2)$$

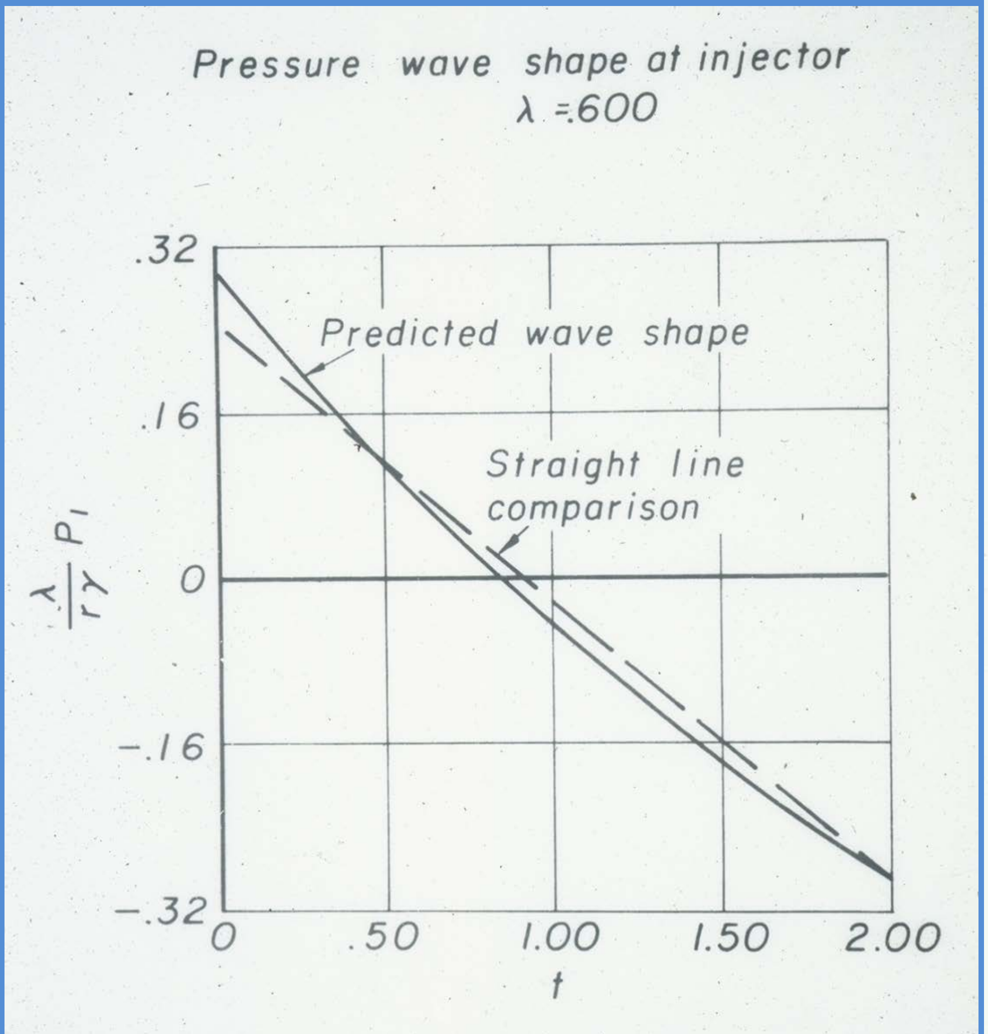
Oscillation Period

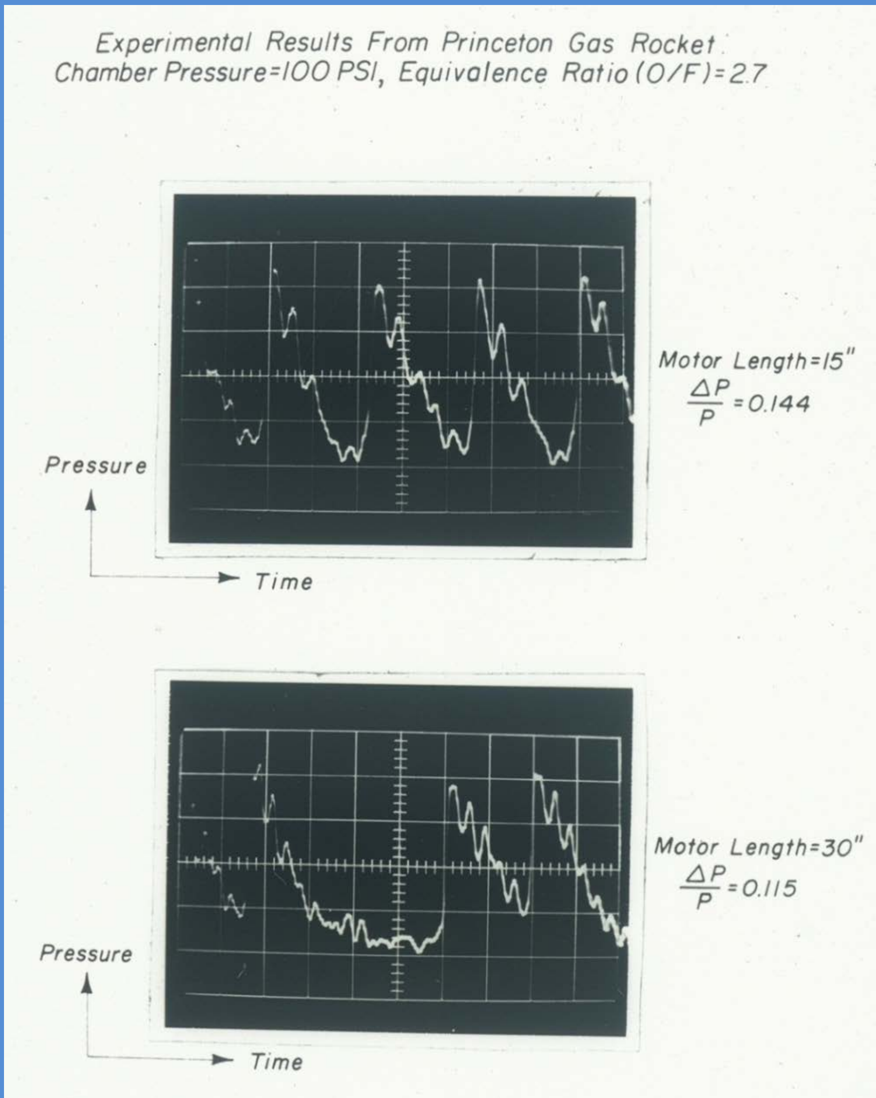
$$T = \frac{2}{1-u_0^2} + \varepsilon\,\frac{3-\gamma}{4}\,\frac{r}{\lambda}\left[\frac{1+\nu}{1-\nu}\left(\frac{1}{1-u_0}\right)^2 + \left(\frac{1}{1+u_0}\right)^2\right]\left\{\frac{2}{\lambda}\,\frac{1-e^{-\lambda}}{1+e^{-\lambda}} - 1\right\} + O(\varepsilon^2)$$

Pressure and Velocity Waveforms

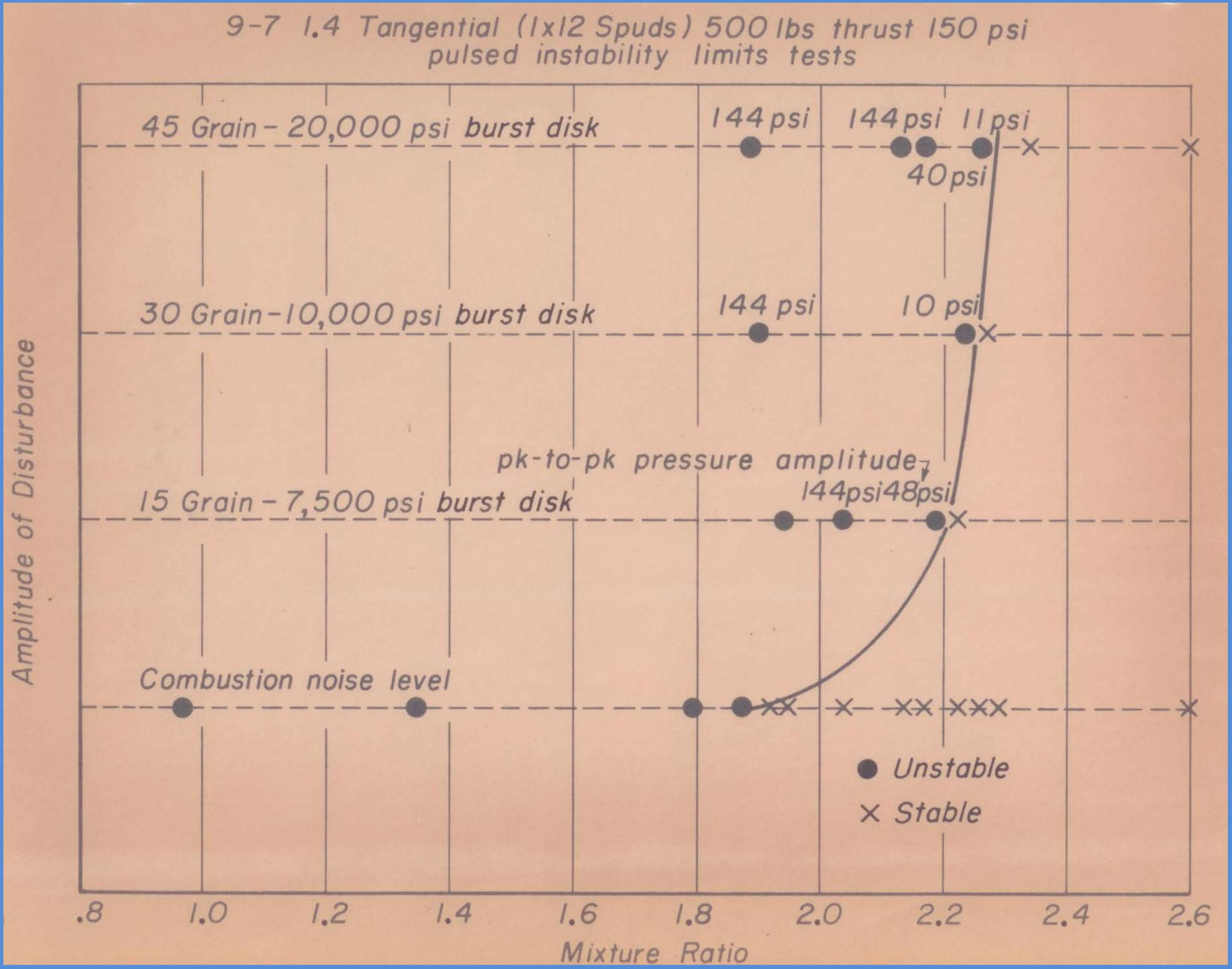


Comparison of Theory and Experiment





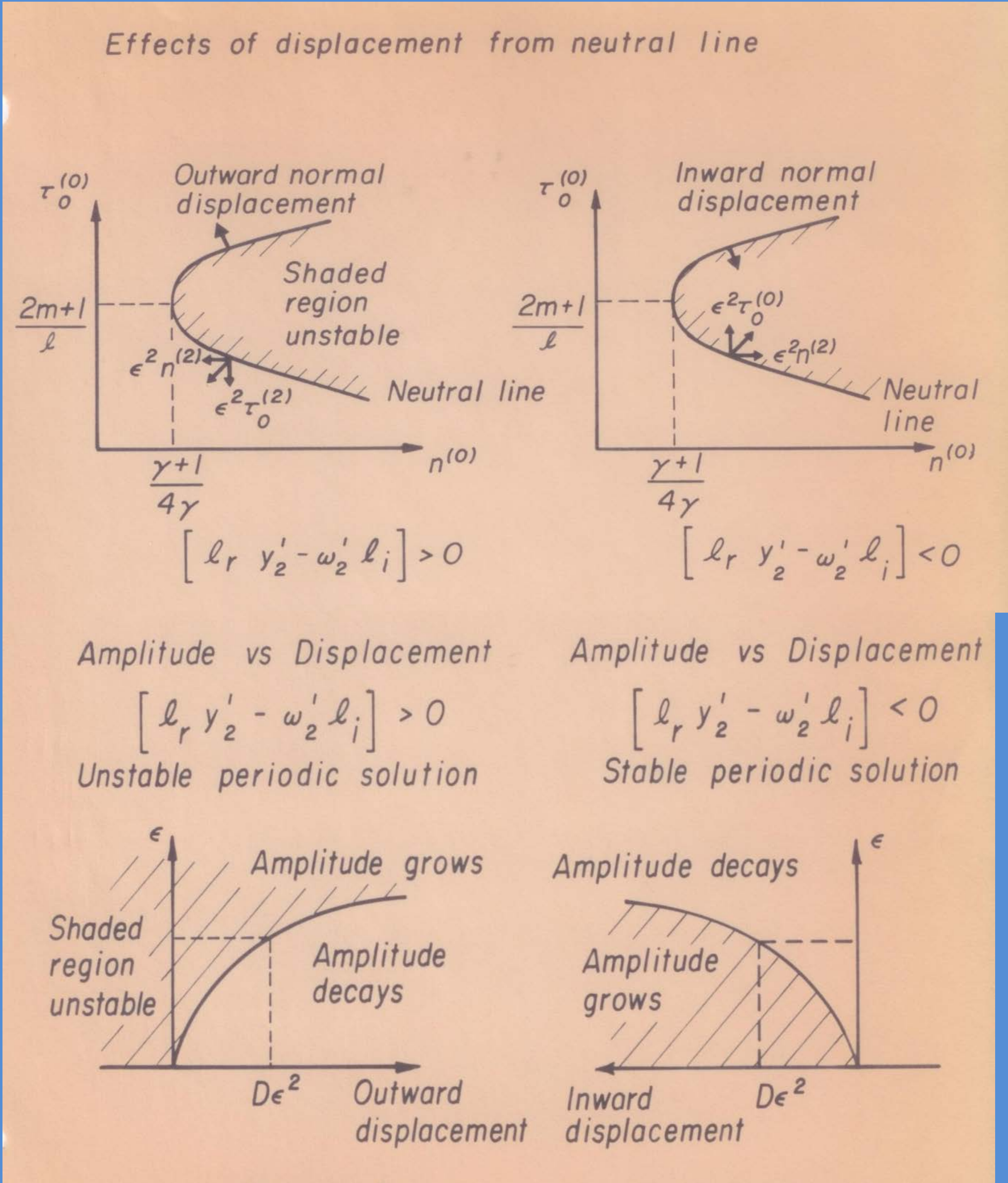
Nonlinear Combustion Instability and Triggering Action



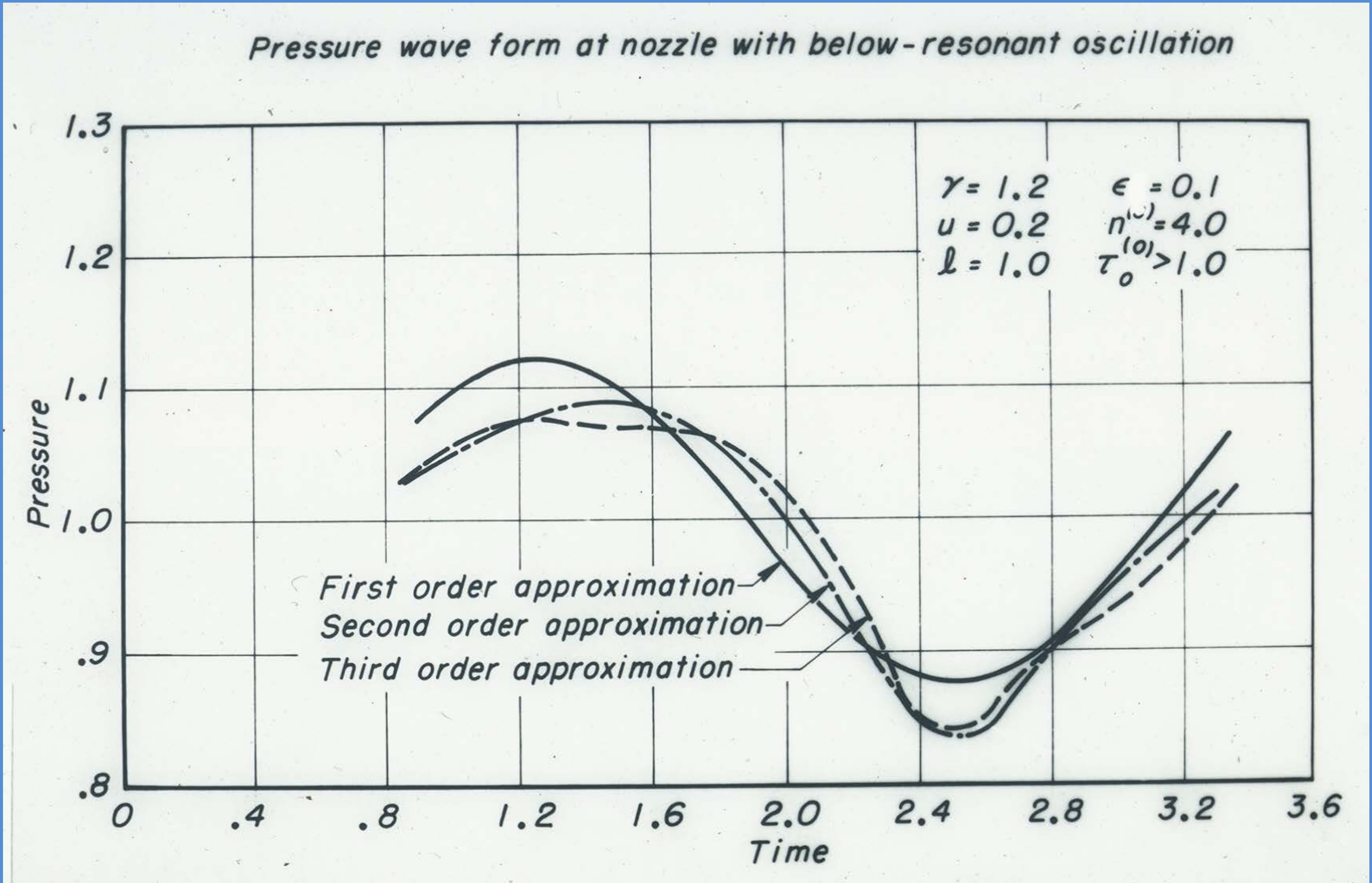
Nonlinear Instability with Time Lag

Perturbation Theory predicts both stable and unstable limit Cycles .

First-order theory predicts linear stability limit while higher-order theory gives a stability surface.



Longitudinal Mode Oscillation without Shock Waves



Liquid-Fueled Ramjet Combustion Instability with vaporization, mixing, and kinetic modelling

Computational Domain, Dimensions, Conditions

- ONE-DIMENSIONAL APPROXIMATION OF A RAMJET COMBUSTION CHAMBER, CONSTANT CROSS-SECTIONAL AREA
- FLOWFIELD: SUBSONIC, COMPRESSIBLE AND NO WALL INTERACTIONS
- EULERIAN (GAS-PHASE) - LAGRANGIAN (LIQUID-PHASE) APPROACH

COMBUSTOR LENGTHS (L)	=	25, 50, 100, 150	cm
INLET GAS VELOCITY	=	5000	cm/s
INLET LIQUID VELOCITY	=	2000	cm/s
INLET GAS TEMPERATURE	=	1000	K
INLET LIQUID TEMPERATURE	=	350	K
INLET GAS PRESSURE	=	10	atm
EXIT MACH NUMBER	=	0.08	

Gas-Phase Equations

- UNSTEADY, SUBSONIC, COMPRESSIBLE, AND EXCHANGING MASS, MOMENTUM AND HEAT TRANSFER WITH THE LIQUID-PHASE.
- EULERIAN APPROACH BASED ON WESTBROOK AND CHASE (1977), EQUATIONS INCLUDE CONTINUITY, MOMENTUM, CHEMICAL SPECIES, ENERGY AND AN EQUATION OF STATE.

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) = \frac{\partial}{\partial x}(\tau \frac{\partial \rho}{\partial x}) + S_\rho$$

$$\frac{\partial}{\partial t}(\rho u) + \frac{\partial}{\partial x}(\rho u^2) = -Eu \frac{\partial}{\partial x}(p + Q) + S_u$$

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} = (1 - \frac{1}{\gamma^*}) \frac{1}{\rho C_p} (\frac{\partial p}{\partial t} + u \frac{\partial p}{\partial x}) + \frac{1}{\rho C_p} \frac{\partial}{\partial x} (\lambda_g \frac{\partial T}{\partial x}) + S_T$$

$$\frac{\partial}{\partial t}(\rho Y_i) + \frac{\partial}{\partial x}(\rho Y_i u) = \frac{\partial}{\partial x}(\rho D \frac{\partial Y_i}{\partial X}) + S_{Y_i}$$

$$p = \rho T / \overline{M}$$

where $\tau \equiv$ artificial diffusivity coefficient,

and $Q \equiv$ artificial viscosity.

$$Q = -\beta \frac{\partial u}{\partial x}$$

$$Eu = p^* / (\rho^* u^{*^2})$$

Liquid-Phase Equations

- LAGRANGIAN FORMULATION IS BASED ON THE EFFECTIVE CONDUCTIVITY MODEL OF ABRAMZON AND SIRIGNANO (1989).

$$\frac{dx}{dt} = U_l$$

$$\frac{dU_l}{dt} = \frac{3}{16}\frac{\mu}{\rho_l}\frac{(U - U_l)}{r_l^2} Re C_D$$

$$Re = \frac{2\rho \mid U - U_l \mid r_l}{\mu}$$

$$C_D = \frac{24}{Re}[1 + \frac{Re^{2/3}}{6}]$$

$$\frac{dr_l}{dt} = -\dot{m}_l/(4r\rho_l r_l^2)$$

$$\frac{\partial T}{\partial t} = \frac{1}{\rho C_p r^2}\frac{\partial}{\partial r}(k_{eff} r^2 \frac{\partial T}{\partial r})$$

at the droplet center, $r = 0$: $\frac{\partial T}{\partial r} = 0$

at the droplet surface, $r = r_l(t)$: $\frac{\partial T}{\partial r} = \frac{Q_l}{4\pi k_l r_l(t)^2}$

$$\frac{k_{eff}}{k_l} = 1.86 + tanh\left[2.245\, log_{10}(\frac{Pe_l}{30})\right]$$

$$C_{10}H_{22} + 15.5O_2 \longrightarrow 10CO_2 + 11H_2O$$

$$W'_{CH} = A\left(\frac{Y_f \rho'_g}{M_{f'}}\right)^a \left(\frac{Y_o \rho'_g}{M'_o}\right)^b exp(-E_A/RT)$$

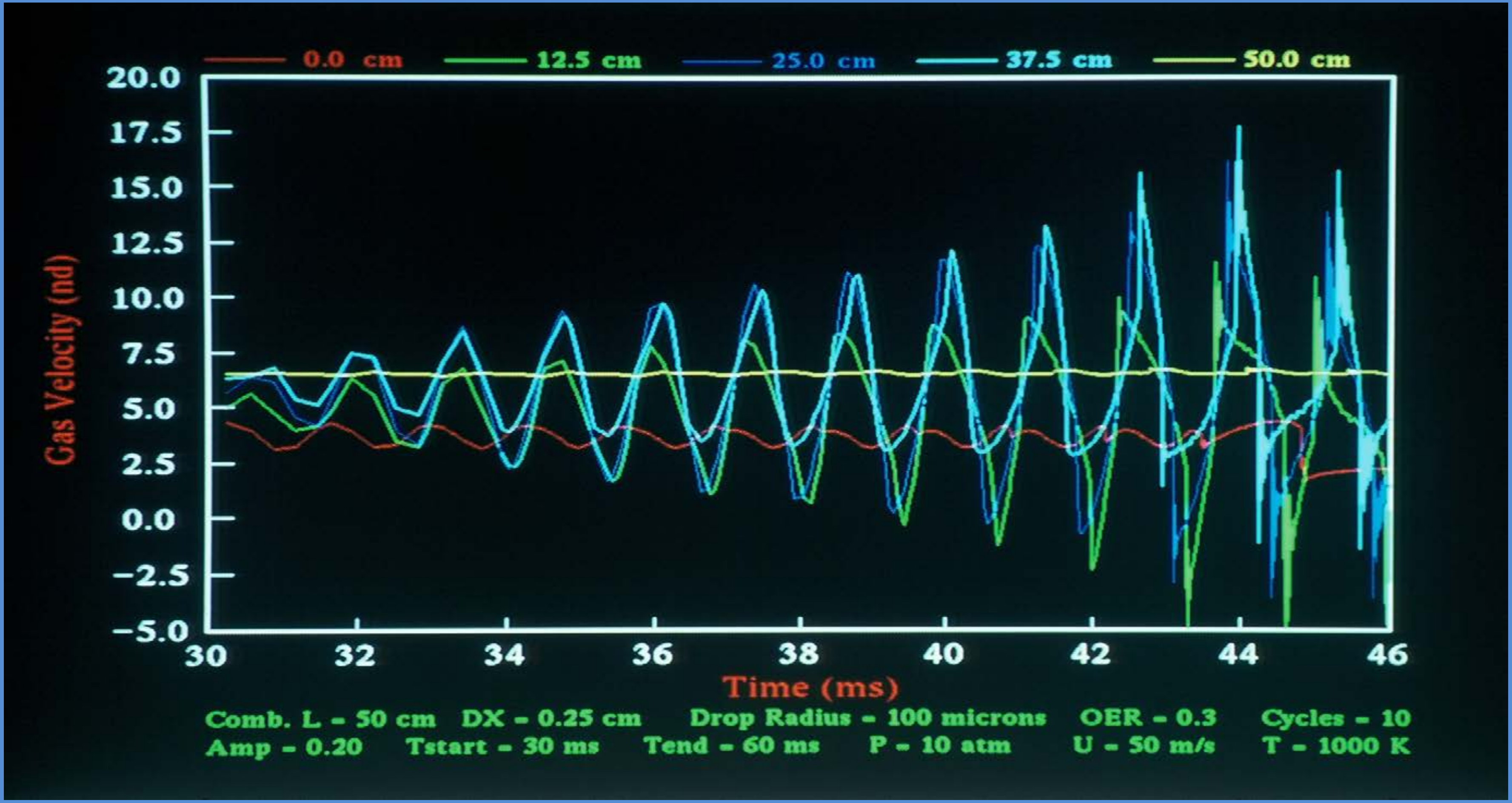
Initial and Boundary Conditions

- DIRICHLET BOUNDARY CONDITIONS ARE USED AT THE COMBUSTOR INLET, WHERE MASS FLOW (TIME-DEPENDENT), GAS TEMPERATURE, AND SPECIES CONCENTRATION ARE SPECIFIED.
- THE PRESSURE AT THE INLET IS DETERMINED BY TAKING THE STAGNATION PRESSURE THERE TO BE CONSTANT.
- OVERALL EQUIVALENCE RATIO IS SPECIFIED AT THE INLET.

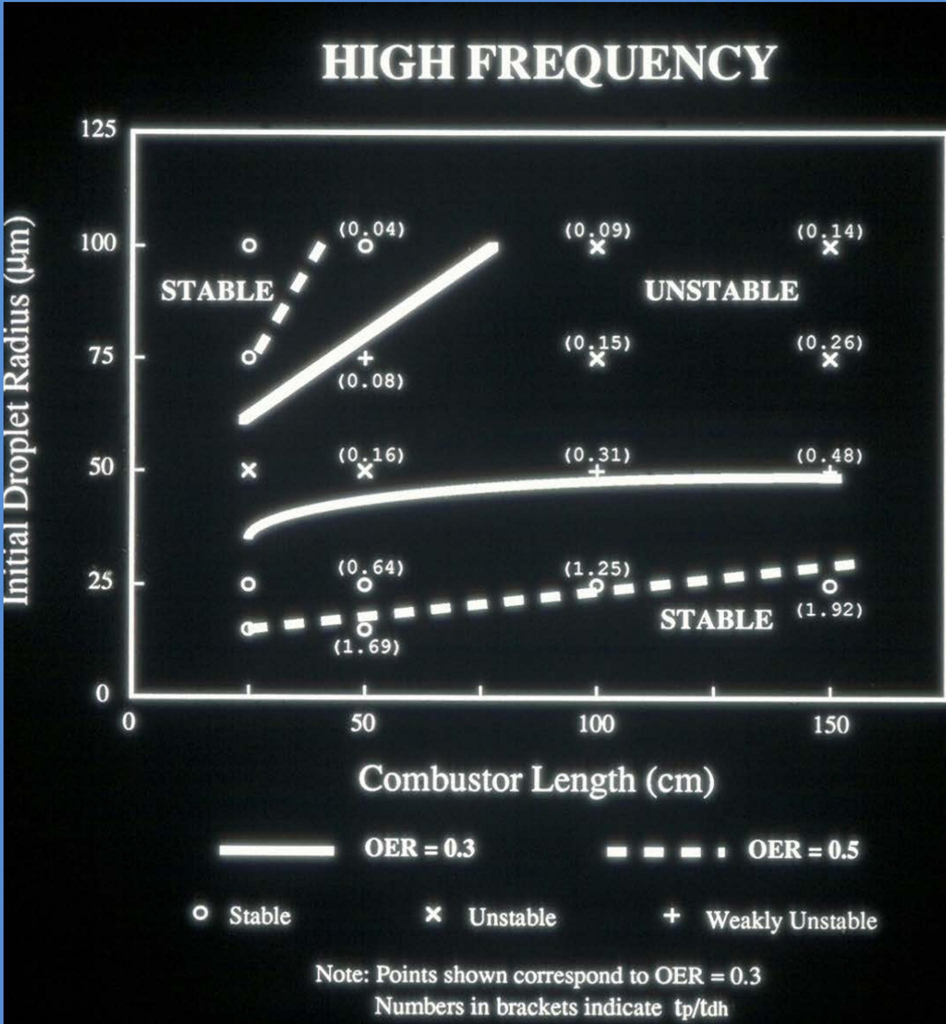
- AT THE OUTLET NEUMANN BOUNDARY CONDITIONS ARE USED, WHERE ZERO GRADIENTS OF THE DENSITY, TEMPERATURE AND SPECIES CONCENTRATION ARE EMPLOYED.
- THE SHORT-NOZZLE APPROXIMATION AT THE COMBUSTOR OUTLET (INLET TO THE NOZZLE) ALLOWS THE MACH NUMBER AT THE EXIT TO BE CONSTANT (CROCCO AND SIRIGNANO (1966))

- INITALLY THE GAS-PHASE TEMPERATURE, VELOCITY, PRESSURE AND SPECIES CONCENTRATION ARE SPECIFIED. FOR THE LIQUID-PHASE THE DROPLET RADIUS, LOCATION, VELOCITY AND TEMPERATURE ARE SPECIFIED.

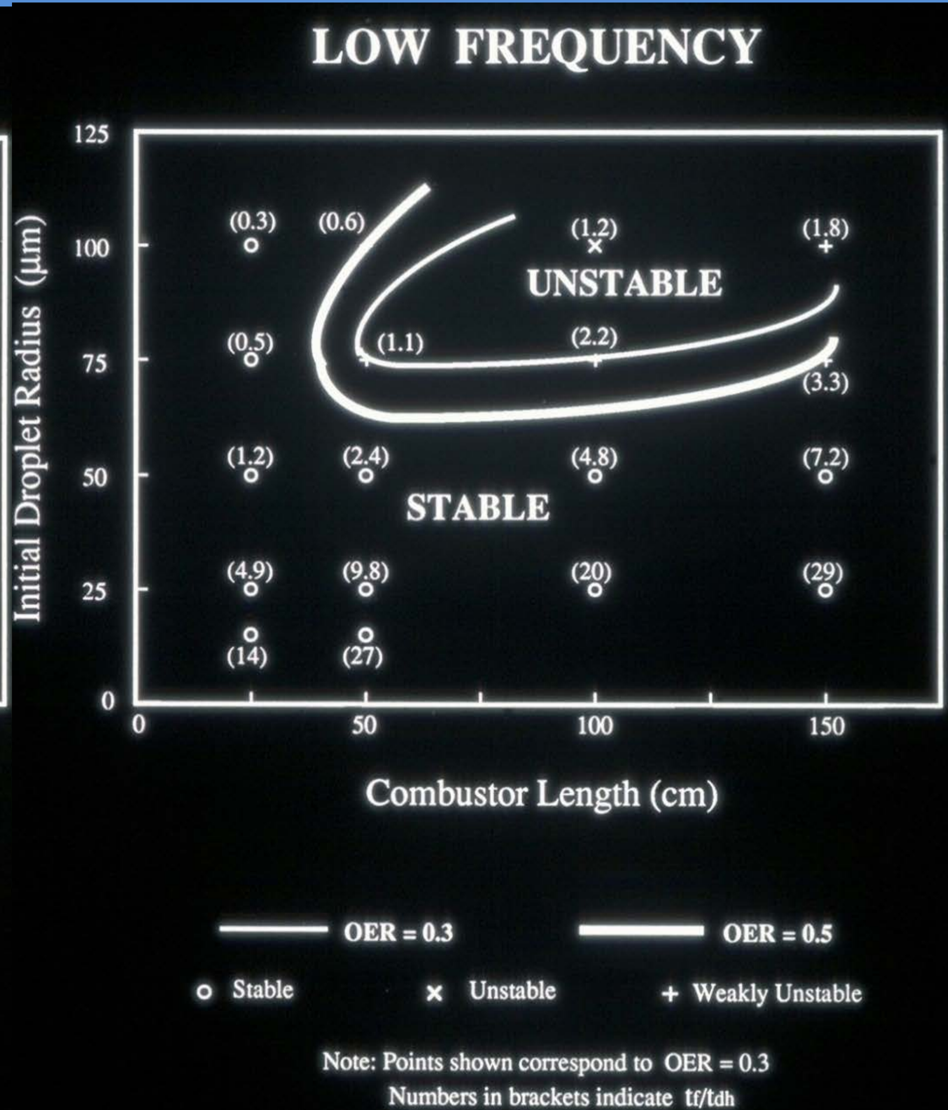
Instability is predicted



Ramjet Instability Domains



$$\tau_{dh} \sim (\rho_l C_{p_g}/\lambda_g) r_l^2 \approx d_l^2/1.22 * 10^{-06}$$



$$\tau_f = 2L/u_{g,inlet}$$

Observations (1)

- THREE DISTINCT FREQUENCIES OF OSCILLATIONS, WHICH MAY EXIST SIMULTANEOUSLY, ARE OBSERVED.
  - THE ONE MOST PREVALENT IS NEAR THE ACOUSTIC FREQUENCY OF THE COMBUSTOR, WHICH IS OF ORDER OF 800 Hz.
  - A LOWER FREQUENCY OF ABOUT 100 Hz IS ALSO OBSERVED FOR LARGER, i.e. 75 AND 100 μm DROPLETS.
  - AT HIGHER EQUIVALENCE RATIOS WE SEE FREQUENCIES OF ORDER OF FEW THOUSAND Hz, WHICH CORRESPOND TO THE ARTIFICIAL INJECTION FREQUENCY OF THE DROPLETS.

Observations (2)

- VAPORIZATION IS RATE CONTROLLING. THE COMBUSTION PROCESS CAN OCCUR IN EITHER A STABLE OR UNSTABLE MODE.
- LOW FREQUENCY OR ENTROPY OSCILLATIONS RESULT WHEN THE CHARACTERISTIC TIME FOR DROPLET HEATING IS CLOSE TO THAT FOR THE GAS RESIDENCE.
- HIGH FREQUENCY OSCILLATIONS RESULT WHEN THE RATIO OF CHARACTERISTIC TIME FOR PERIOD OF OSCILLATION TO DROPLET HEATING IS CLOSE TO 0.15.
- THE FREQUENCY OF FREE OSCILLATIONS IS VERY CLOSE TO THE ACOUSTIC FREQUENCY OF THE COMBUSTOR, AT LOWER EQUIVALENCE RATIOS. AT HIGHER EQUIVALENCE RATIOS OVERTONES ARE PRESENT.

Observations (3)

- IN CASES WHERE THE DAMPING OF EXCITED ACOUSTIC OSCILLATIONS OCCURS, THE EXCITATIONS TEND TO DAMPEN IN ABOUT TEN CYCLES.

- THE DROPLET SIZE, FOR THE SAME EQUIVALENCE RATIO AND COMBUSTOR LENGTH, HAS A PROFOUND EFFECT ON THE COMBUSTOR STABILITY. A RANGE OF DROPLET RADII FOR WHICH THE COMBUSTION IS UNSTABLE IS OBSERVED.

- AN INTERMEDIATE RANGE ALSO EXISTS FOR COMBUSTOR LENGTH, LIKE THE DROPLET RADII, WHERE THE COMBUSTOR OPERATION IS UNSTABLE.

- INCREASING EQUIVALENCE RATIO ENLARGES THE DOMAIN OF UNSTABLE OPERATION.

Thank you.